Percolating Swarm Dynamics

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Abstract. Swarm Intelligence has been a successful approach to solve some combinatorial problems through the metaphor of interacting evolving individuals of a a population P in a closed torus-like space S. Each individual usually perceives an space sorrounding it which can be generally modelled as disk of radius R. In this paper we discuss that Percolation conditions can be key to allow convergence to reach the optimal results of these kind of systems.

1 Introduction

In a physical sense, Percolation is the slow flow of fluids through a porous media, but in a mathematical setting it means the connectivity among random systems that can be layered over a spatial lattice. Basically, it deals with the thresholds on parameters that set conditions on the spatial connectivity and may allow or hinder complete connectivity [1,9,8]. Here we will look for such connectivity conditions in the framework of Swarm Intelligence (SI), trying to ascertain wether they may be the key to SI success and performance.

We find that there were three main approaches to apply Cellular Automata to evolutionary simulations. In the first approach, Cellular automata cells are the proccessing units, fixed in the nodes of a spatial lattice. In the second approach, the processing elements are agents that can move in the space without physical connections, Reynold's model of *Boids' steering behaviors* [11,12]. Particle Swarm Optimization approaches [7] showed that steering behaviors together with a memory of visited positions that maximize a local cost function are enough to solve combinatorial optimization problems. The third approach, inspired in the collective behavior of social insects are the so called *ant colonies* [3,5,6,4]. Ant colonies are strongly cohered by a kind of indirect communication called Stigmergy, that consists in leaving tracks of pheromone that subsequently atract other individuals in a positive feeback. The behaviour of Ant Colonies can be interpreted as intelligent, searching for optimal solutions to combinatorial optimization problems.

In a previous work [2] we found that a simple SI model based on *boid's steering behaviours*, with only the ability to distinguish friend from foe, is able, under the appropriate metaphorical interpretation, to solve efficiently the problem of coloring graphs[10]. Now the research question we address in this paper is wether the convergence can be affected through simple adjustments of the relationship between world size and individual boid perception spatial parameters.

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Our hypothesis is that the concept of Percolation, will give answers to this question and that these answers may give hints to its generalization to other spatial SI systems. We will apply the model of continumm disk Percolation to find the relation between the radius of vision of boids, the size of the world and the size of the population that ensures probabilistically the interaction of boid individuals within the flock. We show that there is a Percolation threshold for these parameters that affect convergence of the SI to optimal solutions of the metaphorical problem.

Section 2 gives some intuition and ideas about Percolation. Section 3 reviews the definition of a SI based on Boids steering behaviors. Section 4 gives computational solutions to the Percolation problem in the Boids system that is a necessary condition for convergence of the system to the optimal solution. Section 5 gives a final account of our conclussions.

2 Some Ideas about Percolation

The Percolation [1,8] concept comes from chemistry and refers to the way a liquid flows through a porous medium. A model of this phenomenon is the *bond Percolation*: given a square lattice in $\mathbb{Z}^2$, assume that we prune the link connecting a position(x, y) to an adjacent one (x', y') with probability $1 - p$, and we mainting it with probability p, defining in this way a random graph. The the critical value p_c that ensures the connectivity from one end of the lattice to the opposite for values $p \geq p_c$ is the Percolation threshold.

The concept of *site Percolation* puts emphasis on nodes and not in links. Imagine an electrical potential from one side to the opposite side of the grid. We started to cut the network nodes thereby preventing electrical current flow. What percentage of nodes should be maintained for the current to continue flowing?. The Percolation threshold p_c is the mean of that measure over all the possible grids.

At the Percolation threshold, the structure changes from a collection of many disconnected parts to a large aggregate. If the grid were infinite, an infinite cluster should be produced and this is the mathematical base of Percolation models. At the same time, the average size of clusters of finite size that are disconnected to the main cluster, decreases. Let p be the probability that a site or a link (depending on the model used) is open. The probability of a site or link belonging to the infinite cluster is:

$$\theta(p) \propto (p - p_c)^{\beta} \tag{1}$$

with $\beta = 5/36$ for 2D networks. Symbol $\propto$ denotes proportionality.

3 Boids SI Parameters

In Craig Reynold's Boids systme [11,12] the board is a continuous domain, and boids are agents that can change their positions in the world. Each individual boid exhibits a very simple behavior that is specified by a few simple rules that guide it to get along with the collective motion of the flock. The global behavior of the flock emerges from the individual boid decisions. The system parameters are:

Position and velocity: Given a set of n boids, the steering rules for i-th boid b_i, at time instant $t+1$ are defined as a function of the position p_j and the velocity v_j of the neighboring boids at the previous instant t. Velocity vector $v_i = (x, y)$ has an angle γ_{v_i} relative to the horizontal axis.

Sensorial input: Each boid is aware of an spatial region around it, its neighborhood. The angle of vision, denoted ρ, comprises all the positions in the plane

$$p = p_i + v' \tag{2}$$

where

$$\gamma_{v_i} - \frac{\rho}{2} \leqq \gamma_{v'} \leqq \gamma_{v_i} + \frac{\rho}{2}. \tag{3}$$

In this article, the value $\rho = 360^o$, meaning that individuals can even observe what happens behind their backs. Let R be the radius of vision: the spatial neighborhood is the disk sector of radius R and angle ρ around the boid. Let ∂_i denote the boids population in the spatial neighborhood. The set of boids lying inside the neighborhood of the i-th individual is denoted:

$$\partial_i = \partial(b_i) = \{b_j : \|p_i - p_j\| < R\}. \tag{4}$$

Effectors: The steering basic rules, used in our model, are the classical ones of the Reynolds model: alignment, separation, and cohesion. Combining these rules, the boids are able to flight co-coordinately avoiding collisions. The flocking rules for the boid b_i are formalized as follows:

- **Separation**: steer to avoid crowding local flockmates inside a private zone of radius z.

$$v_{s_i} = - \sum_{b_j \in \partial_i : d(b_j, b_i) < z} (p_j - p_i) \tag{5}$$

- **Cohesion**: steer to move toward the average position c_i of local flockmates

$$v_{c_i} = c_i - p_i \quad \text{where} \quad c_i = \frac{1}{|\partial_i|} \sum_{b_j \in \partial_i} p_j \tag{6}$$

- **Alignment**: steer in the direction of the average heading of local flockmates.

$$v_{a_i} = \frac{1}{|\partial_i|} \sum_{b_j \in \partial_i} v_j - v_i \tag{7}$$

- **Own velocity**: is the current velocity component, an inertia that has the effect of limiting the rotation angle of the object.

$$\alpha_o v_i \tag{8}$$

The motion model of flocking boids was a linear combination of separation, cohesion and alignment . Here $\|p\|$ denotes the norm of a position or vector p and f_{maxv}

is a non-negative parameter that limits the norm (the length in the Euclidean distance) of a vector and $\mathcal{N}(p) = \frac{p}{\|p\|}$ represents the normalized position or vector.

$$p_i(t+1) = p_i(t) + v_i(t+1) \tag{9}$$

A basic flock model in a D-dimensional torus is a tuple $\mathcal{F} = \left(\mathcal{B}, \bar{\alpha}, \bar{\beta}, \bar{v}, \bar{v}(0), \bar{p}(0)\right)$ where:

- $\mathcal{B} = \{b_1, \ldots, b_P\}$is the population of boids, of size $P \in \mathbb{N}$.
- $\bar{\alpha} = (\alpha_o, \alpha_s, \alpha_c, \alpha_a, \alpha_n)$ are the flock parameters.
- $\bar{\beta} = (S, R, \rho, z)$ where $S \in \mathbb{R}^+$ is the size of the world. The world is square, being the side S and the area S^2. It wraps horizontally and vertically forming a torus centered in the null position $\bar{0} \in \mathbb{R}^D$. The radius of vision is R and the angle of vision ρ. We denote the D-dimensional torus $\mathbb{T}^D$.
- The flock system that steers the boids in the in the world is given by the iteration:

$$v(t+1) = f_{\mathrm{maxv}} \mathcal{N}\left(\alpha_o v(t) + \alpha_s v_s(t) + \alpha_c v_c(t) + \alpha_a v_a(t) + \alpha_n v_n\right) \tag{10}$$

  where $v(t) \in \mathbb{T}^D$ $\alpha_o, \alpha_s, \alpha_c, \alpha_a \in [0,1]$ are global parameters for the *o*wn, *s*eparation, *c*oherence and *a*lignment velocities and the *n*oise $\alpha_n v_{n_i}$ is the product of a random normalized vector v_n by a noise parameter α_n. Notation $v_i(t)$ and $v_{x_i}(t)$ with $x \in \{o, s, c, a, n\}$ represent the velocities of boid b_i at instant t.
- The initial conditions for $\bar{v}(0) = (v_1(0), \ldots, v_i(0), \ldots, v_P(0))$ gives the initial velocities to the boids and it is randomly generated for the extent of this work.
- The initial position is a tuple $\bar{p}(0) = (p_1(0), \ldots, p_i(0), \ldots, p_P(0))$.

4 Percolation Thresholds in Boids SI

Convergence of the Boid SI to some stable (and optimal state under some metaphorical interpretation of the system's configuration), needs that all the boids are connected through their sensory input. Here is where the notion of Percolation is relevant to the Swarm Dynamics. Percolation conditions must relate the size of the side of the world square with the boid's radius of vision R and the population of birds $\mathcal{P}$. Each bird has an area of vision (spatial neighborhood) $A = \pi R^2$. We want to ensure that a random initial distribution of the items will cross the world to the opposite side stepping only positions that are some boid spatial neighborhood. Figure 1 shows the spatial neighborhoods of boids put in random positions following an uniform probability. At a radius of vision *R=2*, the most of them are disconected. When radius is *R=5.9*, some large clusters are formed. Near the Percolation threshold, *R=11.9*, there is a single white cluster in the figure.

We want to determine the values of the parameters that ensure the mutual influence of individuals. For this purpose we consider the models of Percolation

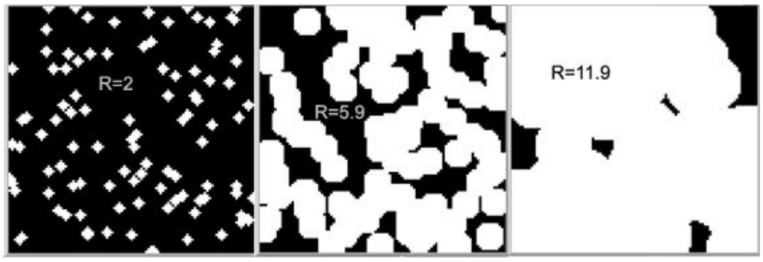

Fig. 1. Spatial neighborhoods of randomly positioned boids for various values of R. Largest radius ensures spatial connectivity.

of continuum systems [1] formed of overlapping $\mathcal{P}$ disks of radius R randomly placed following an uniform spatial distribution in the square world. This is the standard Gilbert disk model or equivalently the Boolean model $G_{R,\lambda}$, used to model an infinite communication network of transceivers that can communicate if they are at euclidean distance at most R. An algorithm to randomly generate such a graph $G_{R,\lambda}$, consists in generating a population of $\mathcal{P}$ disks of radius R whose centers (the nodes of G_R) are selected in the plane $\mathbb{R}^2$ by a two dimensional uniform distribution. After this, nodes at distance lower than R are linked by an undirected edge. The number of points $\mathcal{P}$ is a random variable generated from a Poisson distribution with mean λ that is approximated, for large values of λ by a normal distribution of mean λ and and standard deviation $\sqrt{\lambda}$ given by:

$$P_\lambda(n) = P(\mathcal{P} = n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{11}$$

An square world of side S is divided in S^2 patches of side 1 and a population of $\mathcal{P}$ individuals is distributed over it. Is the distribution randomly created?. Let $\lambda = \frac{\mathcal{P}}{S^2}$ be the mean number of individual per patch. The Poisson distribution predicts that the number of patches with n boids will be $S^2 P_\lambda(n)$ where $n = 0, 1, 2, \ldots, \mathcal{P}$. The problem of Percolation on Gilbert disk model consist in determining if an infinite graph $G_R = G_{R,\lambda}$ generated by a Poisson process is connected. Let $D_{R,\lambda} \subseteq \mathbb{R}^2$ be the set of points over the disks around the nodes of $G_{R,\lambda}$. It is well known that $G_{2R,\lambda}$ has an infinite connected component if an only if $D_{R,\lambda}$ is unbound. A few well-known results on disk Percolation:

– Let x be a node of $G_{R,\lambda}$. The degree of x has a Poisson distribution with mean:

$$a = \lambda A = \lambda \pi R^2 = \frac{\mathcal{P}}{S^2} \pi R^2, \tag{12}$$

therfore, in our problem the number of expected neighbors in the vision area per boid varies in the interval $\lambda \pi R^2 \pm R\sqrt{\lambda \pi}$. Usually, a is called the *degree* of graph $G_{R,\lambda}$, the *connected area* or simply the *area*. Imagine that the spatial neighborhood is filled with white while the background is black. Since $\mathcal{P}\pi R^2$ represents the sum of the areas of vision of all the boids, a is the fraction of white points per patch.

- Since $a > 1$ when $\mathcal{P}\pi R^2 > S^2$, function Φ represents the fraction of a normalized to interval $[0, 1]$, $\Phi_{P,S}$ represents fraction f as a function of the radius R with a population parameter $\mathcal{P}$ and size parameter S. Figure 2 shows a as a function of R in comparison to $\Phi_{100,100}$; note that radius $R = \sqrt{\frac{S^2}{\pi P}} = \frac{10}{\sqrt{\pi}} = 5.642$ sets the limit when area $a = 1$ and white zone is large enough to cover the whole square. Increasing R from that Figure 3 shows $\Phi_{P,S}$ for population of $\mathcal{P} = 100$ boids in a square of side $S = 10$,100 and 1000 patches.

$$\Phi(a) = 1 - e^{-a} \tag{13}$$

$$\Phi_{P,S}(R) = 1 - e^{-\frac{P}{S^2}\pi R^2} \tag{14}$$

- Let $\mathcal{P}_\lambda$ be the set of centers of the disk, we can assume without loss of generality that point $\bar{0} = (0, 0) \in \mathcal{P}_\lambda$. We denote $\theta(R, \lambda) = \theta(a)$ the probability that the connected component to which the origin beongs is infinite. Since $\theta(a)$ is an increasing function, $0 \leq \theta(a) \leq 1$, by the 0-1 law of Kolmogorov, there is a *critical area* a_c such that $\theta(a) = 0$ for $a < a_c$ and $\theta(a) > 0$ for $a > a_c$ and $\theta(a_c) = p_c$ is the Percolation probability threshold. To get bounds to approximate a_c, the easiest way is to compare to well known models based on regular lattices, such as the *face Percolation* (the face is the area of the polygon) on hexagonal lattice where a face is open with probability p and two faces are neighbors if they share an edge. In this way we know that the critical number of neighboring boids in a disk will be limited by:

$$2.184 \leq a_c \leq 10.588 \tag{15}$$

For a population of $\mathcal{P} = 100$ boids in a world of side $S = 100$, from equation 15 we may find bounds on the the radius of vision R using equation 15 as follows:

$$R(a) = \sqrt{\frac{aS^2}{\pi P}} \Rightarrow 8.338 \leq R_c \leq 18.358 \tag{16}$$

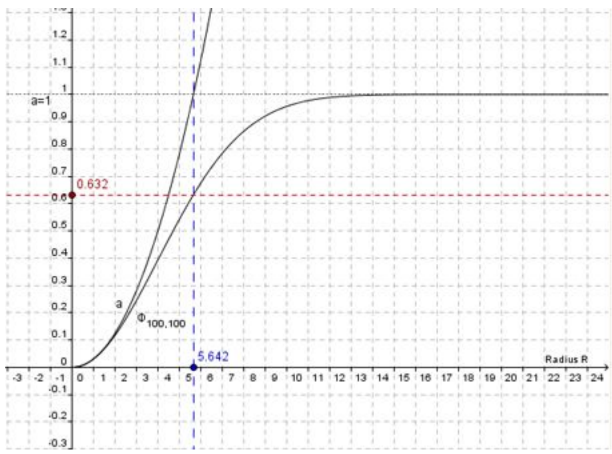

Fig. 2. Percolation: the connected area a as a function of R compared to $\Phi_{100,100}$

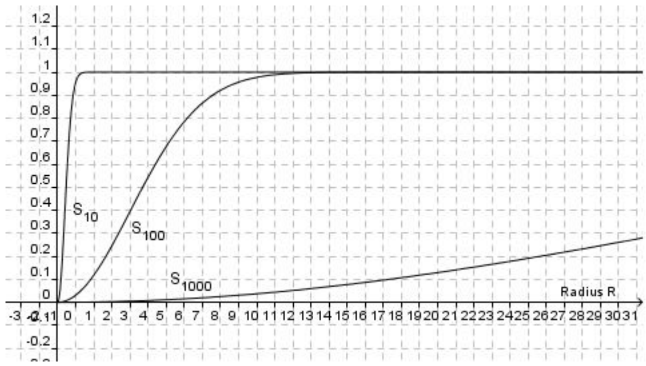

Fig. 3. $\Phi_{P,S}$ for population of $\mathcal{P} = 100$ boids in a square of side $S = 10$,100 and 1000 patches

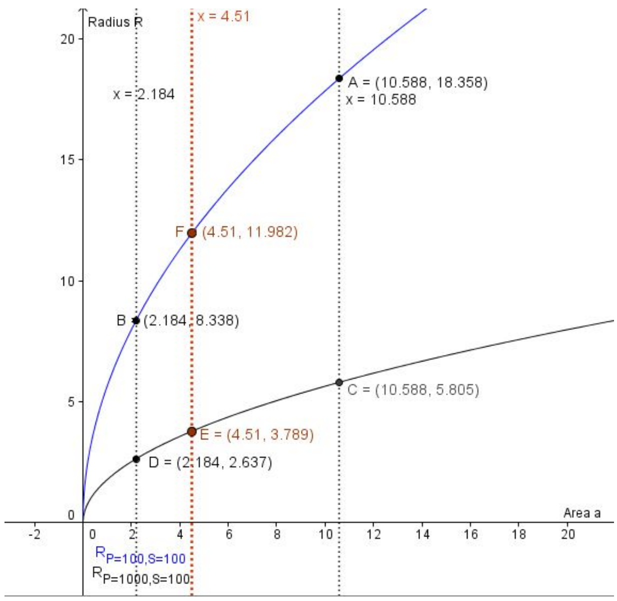

Fig. 4. Variation of the boid's radius of perception versus the area

The curve of R makes the interval decrease as shown in figure 4 when the population P increases and size S remains constant.

For a population of $P = 100$ boids in a square of side $S = 100$ the radius of vision is bounded between:

$$11.979 \leq R_c \leq 11.988 \tag{17}$$

5 Conclusions

Swarm Intelligence can provide good (close to optimal) responses to combinatorial optimization problems through the interpretation of the appropriate metaphor. However, we need also that this response is fast, if it can be of use for something close to practical aplications. In this paper we focus on flocking behaviors because we have shown in a previous paper [2] that this kind of Swarm Intelligence can be applied to find optimal solutions to the problem of graph coloring, improving over other heuristics given in the literature for this problem. Then we feel the strong need to study some convergence requirements. Convergence of the Boids SI needs at least that all the individual boids may somehow sense or be connected to all the other boids. Looking for simple relations among the system parameters that may influence convergence, we have related in this paper the notion of Percolation with the spatial connectivity of the boids provided through their sensing radius. We have provided computational results that show that the general theory of Percolation can actually be used to set the system parameters (sensing radius, space size depending on population size) so that convergence is guaranteed to the (near) optimal solution is minimized.

References

1. Bollobs, B., Riordan, O.: Percolation. Cambridge University Press, UK (2006)
2. Cases, B., Hernandez, C., Graña, M., D'anjou, A.: On the ability of swarms to compute the 3-coloring of graphs. In: Bullock, S., Noble, J., Watson, R., Bedau, M.A. (eds.) Artificial Life XI: Proceedings of the Eleventh International Conference on the Simulation and Synthesis of Living Systems, pp. 102–109. MIT Press, Cambridge (2008)
3. Deneubourg, J.L., Goss, S., Franks, N., Sendova-Franks, A., Detrain, C., Chrétien, L.: The dynamics of collective sorting robot-like ants and ant-like robots. In: Proceedings of the first international conference on simulation of adaptive behavior on From animals to animats, pp. 356–363. MIT Press, Cambridge (1990)
4. Dorigo, M., Blum, C.: Ant Colony Optimization theory: a survey. Theorical Computer Science 344, 243–278 (2005)
5. Dorigo, M., Di Caro, G.: Ant Colony Optimization: A New Meta-Heuristic. In: Angeline, P.J., Michalewicz, Z., Schoenauer, M., Yao, X., Zalzala, A. (eds.) Proceedings of Congress on Evolutionary Computation (CEC 1999), Washington DC, July 6-9. IEEE Press, Los Alamitos (1999)
6. Dorigo, M., Gambardella, L.M.: Ant Colonies for the Traveling Salesman Problem. BioSystems 43, 73–81 (1997)
7. Eberhart, R.C., Shi, Y., Kennedy, J.: Swarm Intelligence, 1st edn. Morgan Kaufmann, San Francisco (March 2001)
8. Grimmett, G.R.: Percolation, 2nd edn. Springer, Heidelberg (1999)
9. Kasteleyn, P.W., Fortuin, C.M.: Phase transitions in lattice systems with random local properties. Journal of the Physical Society of Japan 26(Suppl.), 11–14 (1969)
10. Mizuno, K., Nishihara, S.: Constructive generation of very hard 3-colorability instances. Discrete Applied Mathematics 156(2), 218–229 (2008)
11. Reynolds, C.W.: Flocks, herds, and schools: A distributed behavioral model. Computer Graphics 21 (1987)
12. Reynolds, C.W.: Steering Behaviors for Autonomous Characters (1999), `http://www.red3d.com/cwr/papers/1999/gdc99steer.html`