Q-analysis: an introduction

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- Systems methodologies in social sciences.
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  - Qualitative approach: natural language.
  - Systemic approach: social networks, Forrester diagrams and Q-analysis.
- What is Q-analysis?.
  - Structure: backcloth representation of binary relationships.
  - Function: Traffic over backcloth.
- Can Q-analysis offer a different view?
  - Q-analysis and Complex Systems
  - Breaking Russels paradox: self-organization.
  - Subsume Q-analysis in Lattice computing?

Systems methodologies in social sciences.

Quantitative approach to social sciences research

- Statistical surveys are used to collect quantitative information about items in a population.
- Many phenomena with few variables searching for:
  - Descriptions: means, deviations,…
  - Explanations: factorial analysis, correlation, clustering,…
- Dynamic or not:
  - Time series analysis.

Qualitative approach to social sciences research

- Qualitative research: understanding of human behavior and the reasons that govern such behavior.
  - Investigating the *why* and *how* of decision making, not just *what*, *where*, *when*.
  - Hence, smaller but focused samples are more often needed, rather than large samples.
- A **focus group** (R. K. Merton)
  - A form of qualitative research analyzing the discourse of a group of people.
  - Asked about their attitude towards a product, service, concept, advertisement, idea, or packaging.
- Questions are asked in an interactive group setting where participants are free to talk with other group members.
  - Few cases with a lot of linguistic variables.
  - Need of tools for qualitative data analysis: discrete mathematics, artificial intelligence, computational linguistics.

Systems methodology in social sciences.

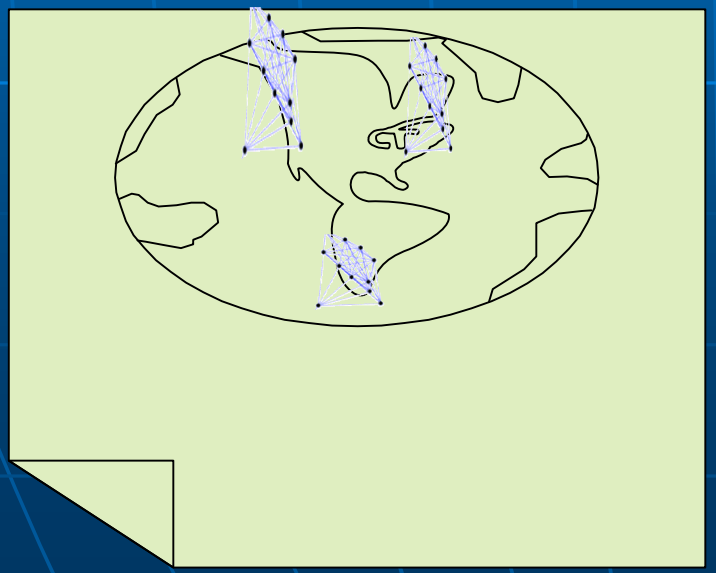
- Systemic approach: Systems have structure, defined by parts and their composition;
- Systems have function: which involves inputs, processing and outputs of material, energy or information,
- Systems have interconnectivity: the various parts of a system have functional as well as structural relationships between each other.

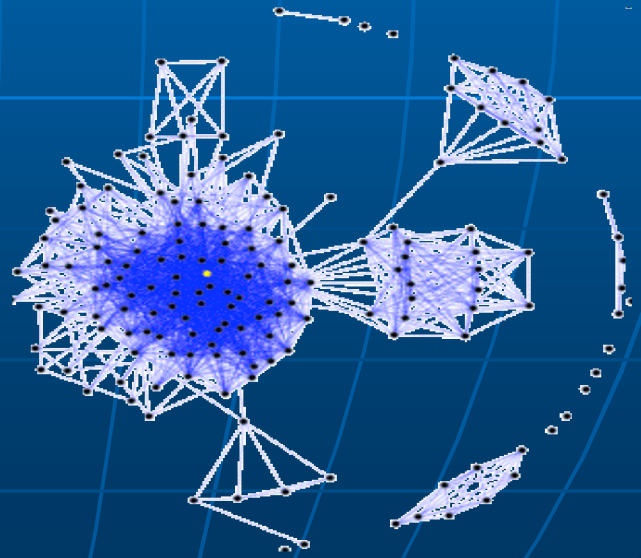
Early systemic approaches to social sciences

- Social Networks: 1950-60
- System Dynamics: 1950-60
- Q-analysis: 1970-80

Social Networks: structural approach

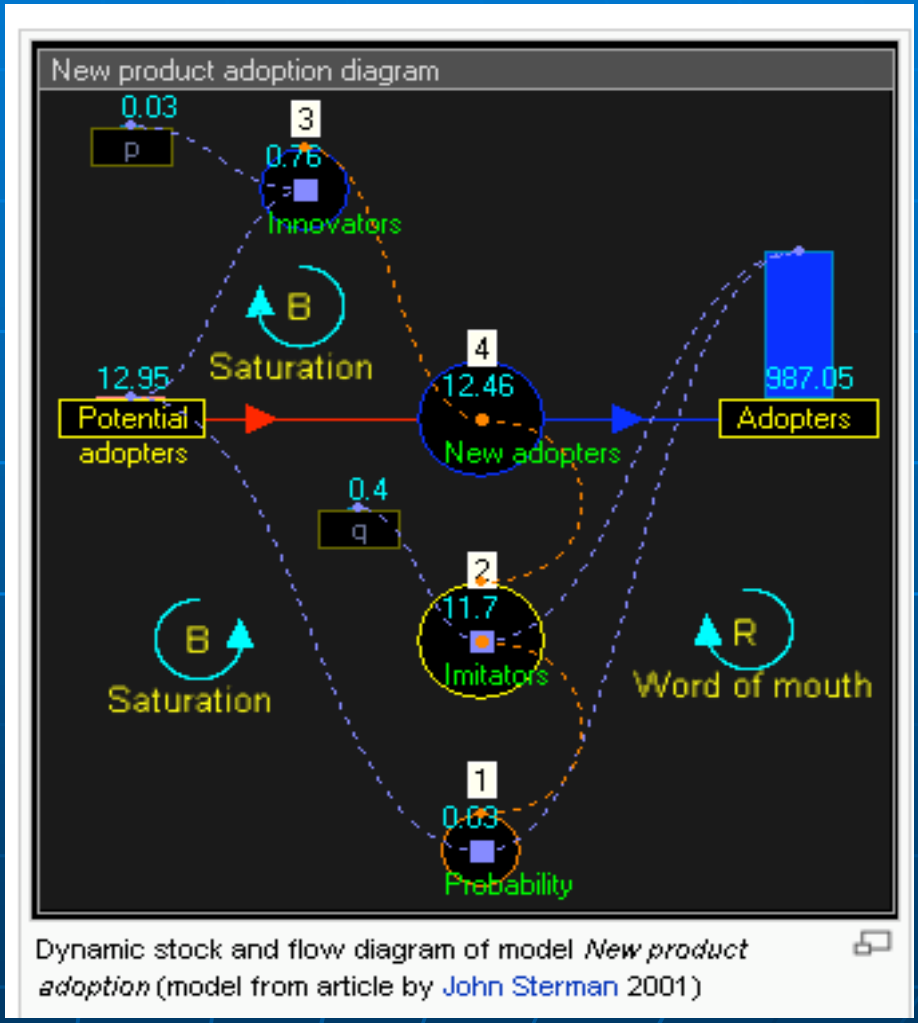
- Graphs made of individuals or organizations connected by one or more specific types of interdependency, such as friendship, kinship, financial exchange, dislike, sexual relationships, or relationships of beliefs, knowledge or prestige.
- In 1954, J. A. Barnes started using the term systematically to denote patterns of ties, encompassing concepts traditionally used by the public and those used by social scientists





System Dynamics: functional approach

- An approach to understanding the behavior of complex systems over time.
- It deals with internal feedback loops and time delays that affect the behavior of the entire system.
- Graphical approach to dynamic modeling based on stocks and flows.
- Was created during the mid-1950s by Professor Jay Forrester of the Massachusetts Institute of Technology.
- Early applications to business management and urbanism.
- Club of Rome 1970: a model of socioeconomic world.



Dynamic stock and flow diagram of model *New product adoption* (model from article by John Sterman 2001)

Q-analysis: semantic approach

- Traffic (function) over a backcloth (structure).
  - Example: categorization of TV programs.
  - Structure:
    - A set of program slots.
    - A set of descriptors of programs contents.
    - Backcloth: A semantic relation.
  - Function:
    - Traffic: share.
  - Changes in structure produce changes in traffic.
  - Changes in traffic produce changes in structure.

What is Q-analysis?

Early Bibliography

- Atkin, R. (1972). From cohomology in physics to q-connectivity in social science. International Journal of Man-Machines Studies vol. 4, 341–362.
- Atkin, R. (1974). Mathematical Structure in Human Affairs. London, Heinemann.
- Atkin, R. (1976). An algebra for patterns on a complex II. International Journal of Man-Machines Studies vol. 8, 483–498.
- Atkin, R. (1977). Combinatorial Connectivities in Social Systems. Basel, Birkhäuser Verlag.

Backcloth

- Graphical representation of a binary predicate as a set of polyhedrons.

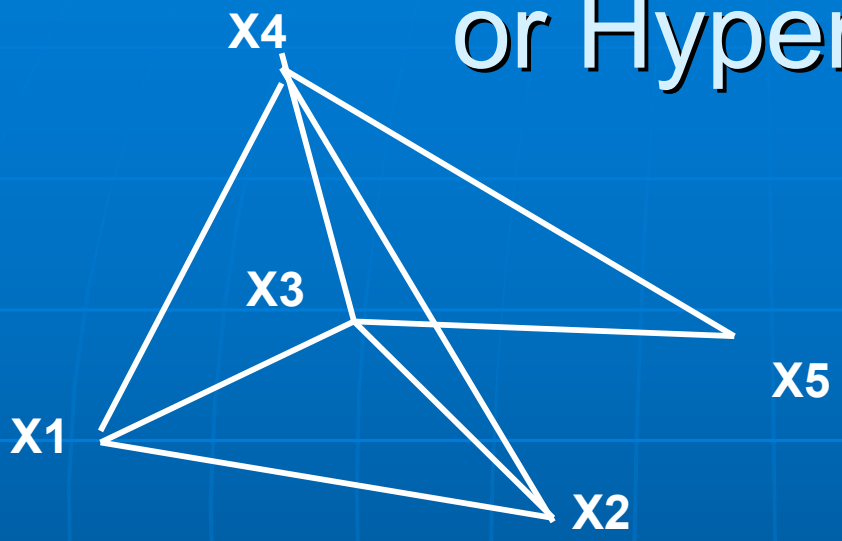
  $P(y_i)=\{x_j: R(y_i,xj)\}$

Descriptors

Items

R	X1	X2	X3	X4	X5
y1	1	1	1	0	0
y2	0	0	1	1	1
y3	1	1	0	1	0
y4	1	0	1	0	0
y5	0	1	1	0	0

Simplicial complex or Hypergraph

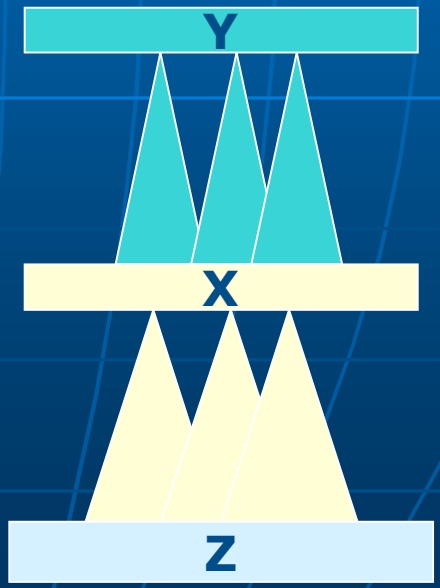


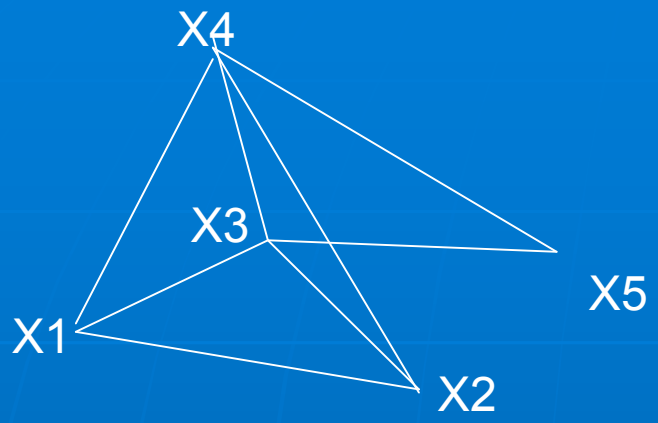
P(y1)={X1,X2,X3}
P(y2)={X3,X4,X5}
P(y3)={X1,X2,X4}
P(y4)={X1,X3}
P(y5)={X2,X3}

The set of descriptors and the set of items should be, following q-analysis, at different levels to avoid Russell's paradox:

$$A=\{A : A \notin A\}$$

Q-analysis deals with structures of levels following type theory





Traffic

P(y1)={X1,X2,X3}
P(y2)={X3,X4,X5}
P(y3)={X1,X2,X4}
P(y4)={X1,X3}
P(y5)={X2,X3}

- **A traffic is a function f(P(yi),t) that associates a value (numerical or logic) to a polyhedron at time t.**
- **Traffic changes the structure and structure determines the traffic: for example, if traffic is 0 the polyhedron disappears.**
- **Interest: transmission of traffic according to a measure of connectivity between polyhedrons**

Traffic(P(yi, t+1))=f(Neighborhood(P(yi)),t)

Definitions

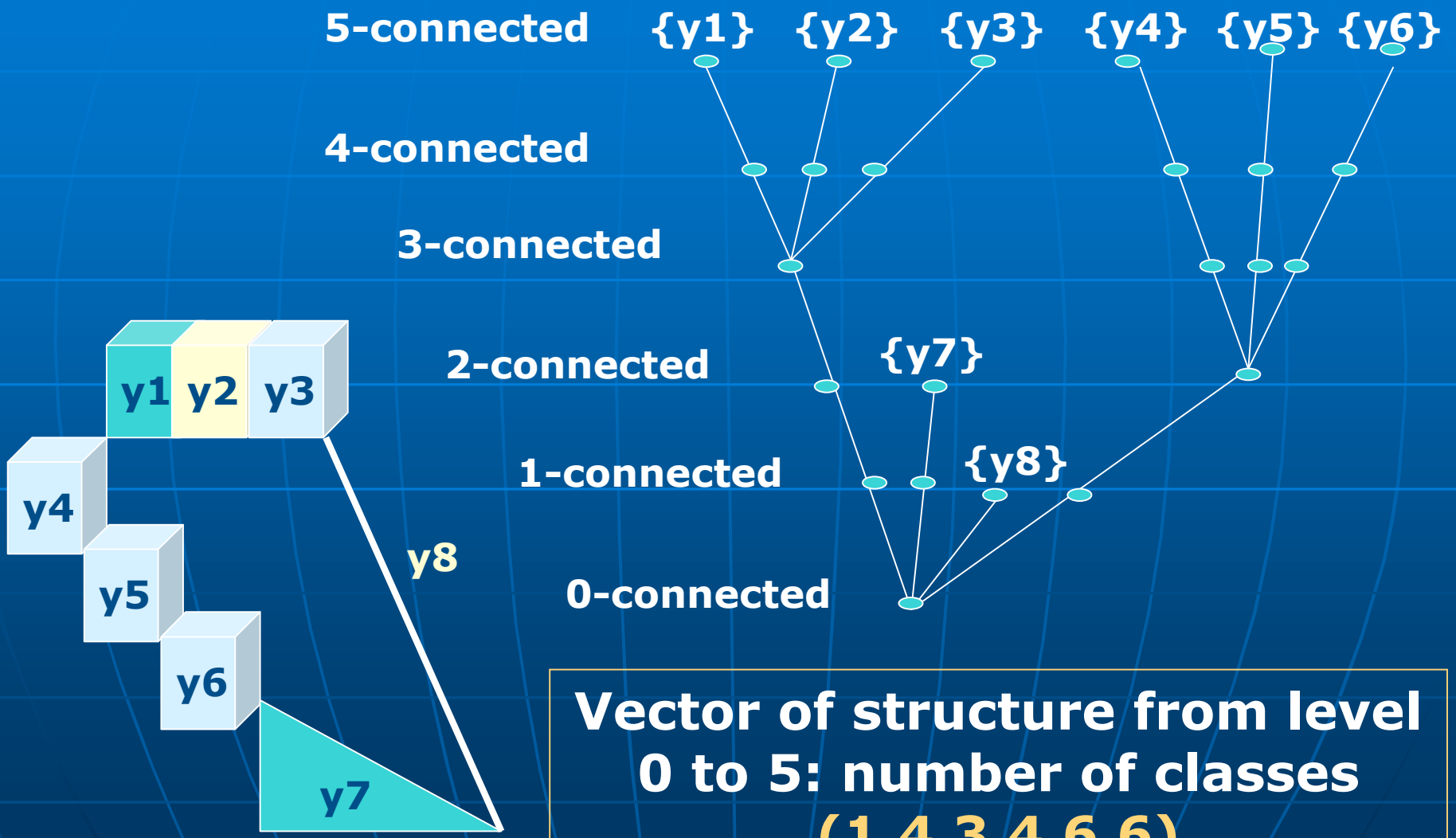
- A descriptor is a point in a multidimensional space.
- A described item is a polyhedron, clique or hypergraph edge.
- We say that two polyhedrons $P(y_1)$ and $P(y_2)$ are q-near if they have a common face of q+1 or more vertices:

$$Dim(P)=\#P-1$$

$$Dim(P(y_1) \cap P(y_2))>=q$$

- We say that two polyhedrons y_1 and y_2, are q-connected if there are joined by a chain of polyhedrons q-near each one to the following.

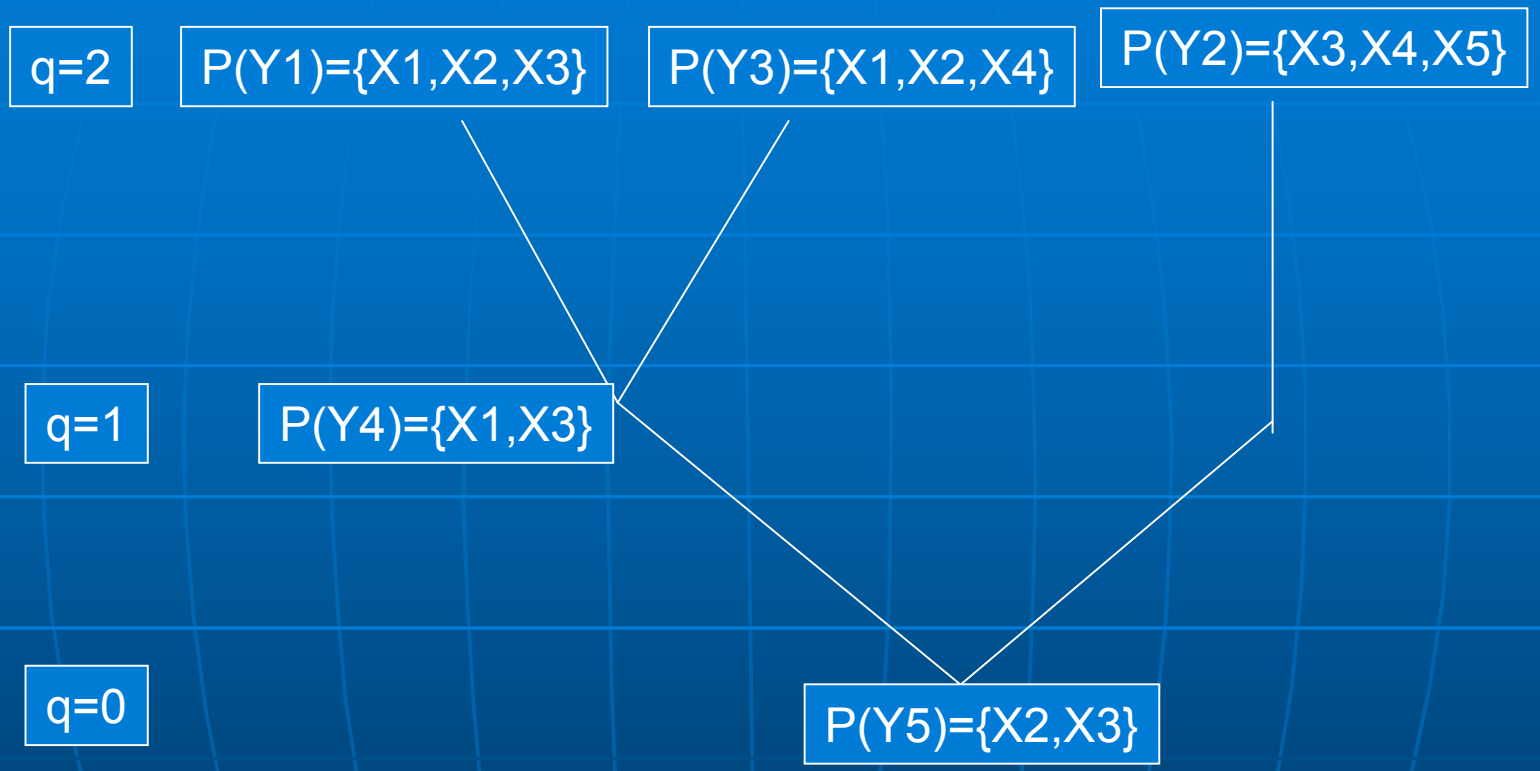
Q-analysis tree



Vector of structure from level 0 to 5: number of classes

(1,4,3,4,6,6)

Previous example



Vector of structure:
(1,2,3)

Can Q-analysis offer a different view?

Q-analysis and Complex Systems

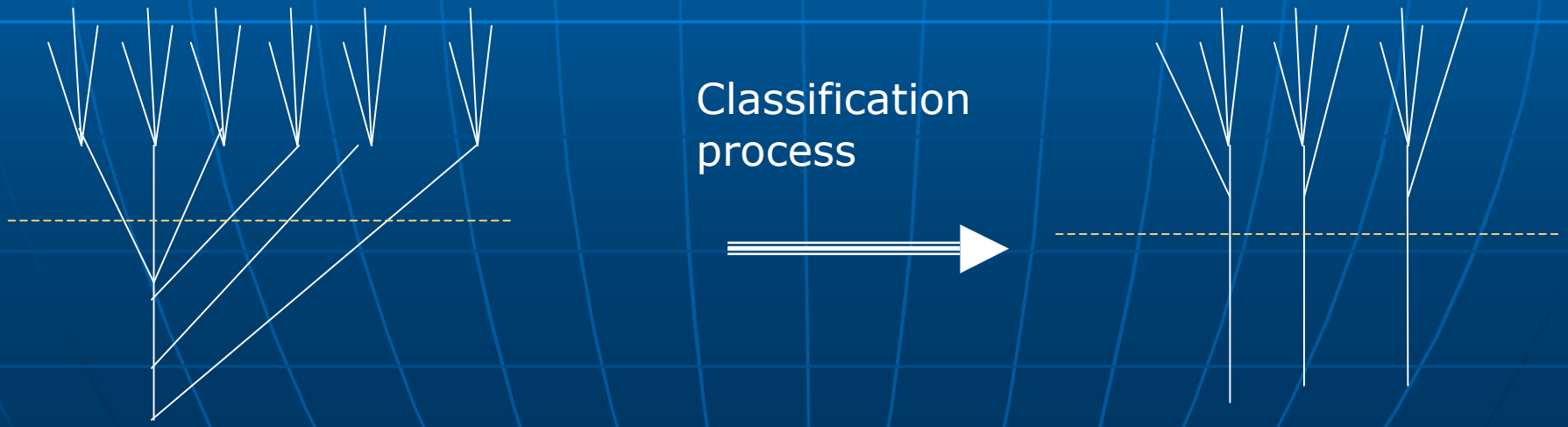
- Vector of structure:
  - At position i, v(i) is the number of equivalence classes for q-connection=q-near* relationship.
  - Allows to compare structural changes along time.
  - The concept of vector of structure can be generalized to other measures of similarity.
- Traffic:
  - Variation in traffic depends on the variation of neighbors: diffusion.
  - The more the connectivity, the more the dependence.
  - Complex systems approach, like cellular automata or neural networks:
    - The board is the structural backcloth.
    - Each polyhedron changes its value according to neighboring values.

How to integrate q-analysis in modern Computational Intelligence?

- Q-analysis was a research line in systems theory as important as evolutionary systems, learning, adaptation… in 1979. But radically declined.
- Proposal? Use Q-analysis to study dynamic clustering, self-organization, evolution: processes that contract distances between near polyhedrons and expand distances to those that are far…
  - In the tree distance based on connectivity.
  - In Euclidean distance since vertices are points.
- For example, on swarm analysis:
  - Think of a vertex as a bird.
  - Think of a polyhedron as the neighbors in a given radius R.

Universal measures

- **Entropy:**
  - **which is the mean level of q-connectivity over all the possible trees given the initial dimensions of polyhedrons?**
  - **How far is a structure from a classification?**
- **Percolation generalized:**
  - **Which is the initial connectivity needed to ensure the mutual influence and hence the convergence of a swarm to a number n of clusters?**



Breaking Russell's paradox: self-organization.

Binary relations on X

- The set of descriptors is equal to the set of items described.
- Example: synonymy. The meaning of a word is a set of words.

$$P(xi)=\{xi1,\ldots,xin\}$$

- Problem: contradiction. Following cross references from white you can reach the word black.
- Ways to avoid contradiction:
  - Connective: Restricting the cross references to those lexical entries P(xj) q-near of P(xi) for q high enough.
  - Self-organizing: Exploiding contradiction to restructure the polyhedrons.

Modeling synonymy Self-organization/Evolution

- Diamond logic: extends boolean logic T, F with a value I of disinformation and a value J of over-information or contradiction.
- Example: synonym dictionaries

P(white)=ashen+blanched+bloodless

P(ashen)=colorless+white + livid.

P(colorless)=neutral+pale+pallid.

P(pallid)=pale+pallid+wan+sick.

P(wan)=sicken+come down.

P(livid)=black-and-blue+ livid.

P(black-and-blue)=purple+violet+purplish

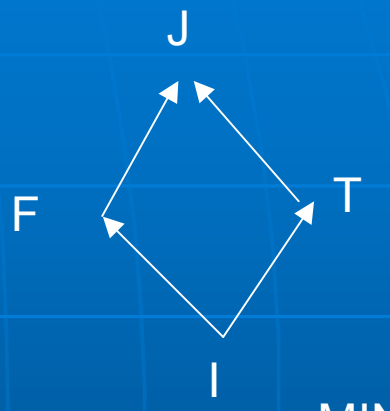
Thesaurus search:

Iteration from initial condition I and white=T and violet=F

Equations are interpreted as Polyhedrons.

Logical values are traffic.

Ashen take value J: contradiction.



Order of Diamond

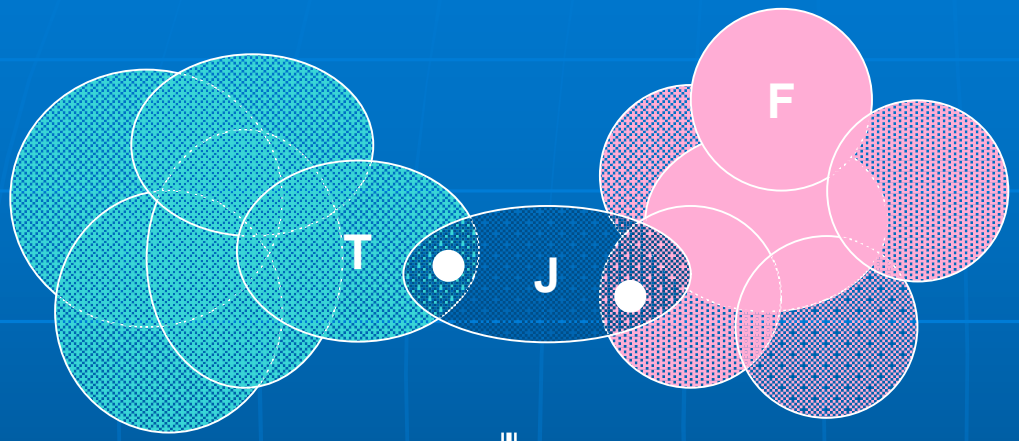
MAX

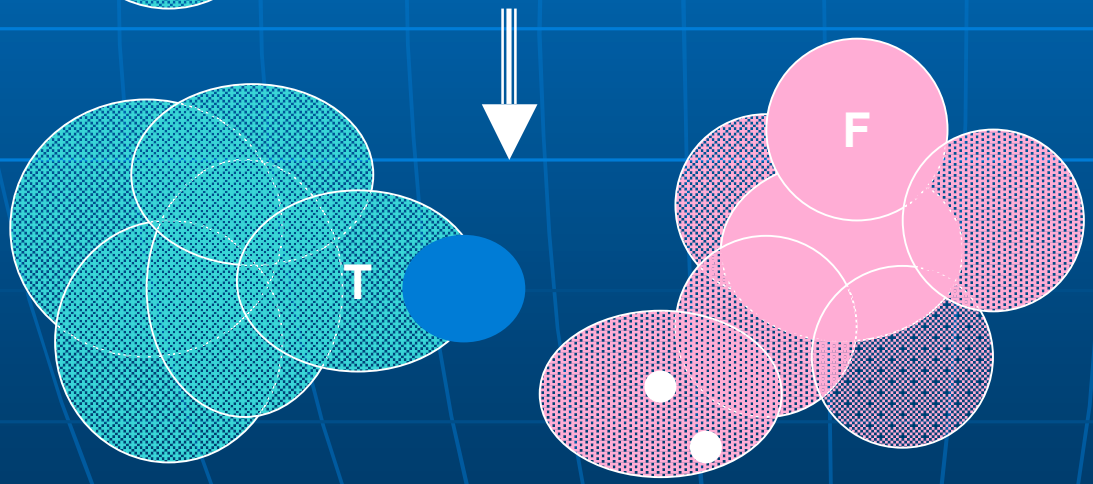
+	T	F	I	J
T	T	J	T	J
F	J	F	F	J
I	T	F	I	J
J	J	J	J	J

MIN

x	T	F	I	J
T	T	I	I	T
F	I	F	I	F
I	I	I	I	I
J	T	F	I	J

Exploit contradiction J to gain information



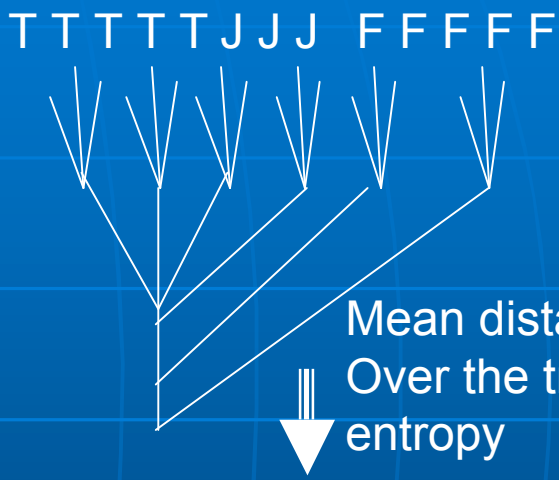


Contract T

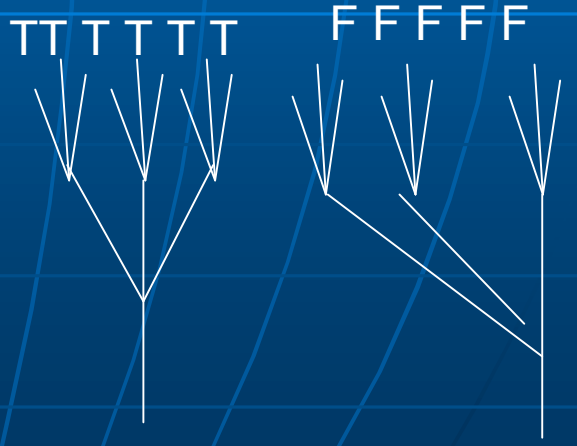
Expand J

Contract F

GENETIC / SELF-ORGANIZING



Mean distance Over the tree= entropy



Research question

- Critics to Q-analysis: less general than clusters analysis.
- Is it interesting nowadays the approach of Q-analysis?
- How can be subsumed Q-analysis in Lattice computing?