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Hierarchically structured systems

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Abstract: Obvious reasons of methodological praxis lead to the fact that in our studies of systems these appear as entities situated in an environment without structure. The hierarchical conception of systems induces a structure in the environment in a natural way making its systemic nature evident, as well as the necessity of its role as controll of the system under study. The functions of optimization of the system's goal are shared by the system itself and by its environment and, in this context, the system can interact in competition or in cooperation. Finally, the hierarchical modelling in System Dynamics, that is, the construction of a set of models of progressively decreasing abstraction, is put forward as a method to enhance the structure of the systems.

Keywords: Hierarchy, philosophy of systems, modelling

1. Introduction

The idea of systems is nowadays totally diffused throughout science, although each discipline uses it with different connotations. The reason for this is that science incorporates as the subject of its study more and more problems of increasing complexity in which the systemic conceptions such as structure, organization, model, information, goal, evolution, etc. appear as central concepts.

To center our discussion, we will take from the literature two definitions of system that we consider in great measure, as complementaries. The first one is that of Bunge (1979):

Definition 1. A system $\boldsymbol{X}$ is defined by the ordered set $\{\boldsymbol{C}(X), \boldsymbol{E}(X), \boldsymbol{S}(X)\}$, where:
– $\boldsymbol{C}(X)$ denotes the composition of $\boldsymbol{X}$, and represents the set of elements which compose $\boldsymbol{X}$.
– $\boldsymbol{E}(X)$ denotes the environment of $\boldsymbol{X}$, and it is the set different things from the components of $\boldsymbol{X}$, which have an action upon then or are under their influence.
– $\boldsymbol{S}(X)$ denotes the structure of $\boldsymbol{X}$, and it is composed by the set of relations (special, functional,...) and links among the elements from $\boldsymbol{X}$, or among the elements from X and elements from its environment.

The second belongs to Klir (1969):

Definition 2. A system $\boldsymbol{X}$ is a set of variations in time of a set of variables under study, considered at a certain level of precision.

Each of them represents one end of the possible definitions of systems. The latter represents the way we perceive the systems, the former reflects the structure and causality we superimpose upon what we perceive in order to explain it. There is, then, on one hand the behaviour without justification and on the other hand the structure from which we can not obtain the behaviour. We need a third definition, a product of the previous ones, which acts as a bridge between them; one dynamical definition which gives an explanation as to how structure generates behaviour. We take again a definition, after Klir (1969), which is of common usage in control and system dynamics theories:

Definition 3. A system $\boldsymbol{X}$ is a set of states and a set of transitions among the states. One can admit a statistical interpretation of the occurrence of the transitions from one state to the other, but it is not necessary to do so.

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In this definition the components of the system are the state variables of the system and the transitions among states are defined by the dynamics relationship among these variables. In this way we relate the behaviour, the change of the states, to the structure of the system, the relationships among the state variables.

The combination of these three definitions provides us with a more complete description of the systems: their components, their interrelation and the way this interrelation produces the temporal behaviour we observe.

1.1. Hierarchical structure

The combined definition of the previous paragraph is very fruitful, but that set of definitions does not separate sufficiently the environment of the system. The neighbourhood of the system remains blurred and unstructured, the only meaningful and structured entity being the system itself. The system is free to arbitrarily act in an environment lacking sense. On the other hand, this environment usually appears as a constraint to the system so that its action should be against hostile surroundings. Let the following definition of System Dynamics due to Coyle (1977) be used as an illustration of what is said.

System Dynamics is: "A method of analysing problems in which time is an important factor, and which involves the study of how a system can be defended against, or made benefit from, the shocks which fall upon it from the outside world".

The hierarchical structure provides us with the adequate frame to focus the systems as taking part in a structured environment. The concept of hierarchical structure has no formal universally accepted definition and we are not going to enter into such discussion, but rather utilize those concepts which serve our purpose.

A survey of our surroundings would reveal to us immediately that not only are systems not isolated, but have strong interactions with other systems and, moreover that, they and their interactions can be considered as being immersed in a system which encompasses them. In this way one arrives quickly at a view of the world as an organic hierarchy of systems. When our goal is the study of a particular problem it becomes obvious that our efforts of structuration and modelling will refer principally to the system within the boundaries that our study presupposes and it would be senseless to model the universe to understand a business company. However, when the interest is in modifying the system, in improving it in any way, we should take into account its environment as something structured which can be affected by our activity. If our observation were a little more accurate we would notice that we have made a lot of simplifications in order to bring out the hierarchical structure of the system. The observation would show us a many-sided world so that each structuration model we assume represents a partial perspective of the complexity of that world. The image that would correspond to that world would be composed of an undefined number of hierarchical structures overlapping in time and sharing their elements.

1.2. Competition and cooperation

When a designer is trying to design a device in a hierarchical manner with modules at different levels, he faces a problem of coordination. The different modules and submodules must be disposed, interrelated and regulated so that their combined action be harmonious and the overall system accomplish some task. The knowledge the designer has, or seeks, about the system is global and he may, without trouble, be ignorant of the inner structure of the elemental modules provided that he knows their fundamental characteristics. The problem to be solved, for each subsystem at each level, is the definition of its appropriate neighbourhood so that the activity of the subsystem be appropriate to its position relative to the global system. The environment in which the global system must be inserted is previously defined and never constitutes a problem the designer must cope with. His responsibility is limited to making the system work as he has been asked so that each module at each level is harmoniously integrated with its environment.

In contrast, for the analyst, in particular for the specialist in social systems, the problem is raised in a different way, nearly in the opposite way. He faces a certain problematic behaviour lacking information about the system and about the bigger systems in which it can be included. His first step will consist in modelling the problematic phenomena and, for him, the environment means no more than the boundary of his model. Then, his work will revolve around the search for an optimum behaviour of the system as an isolated one. It is in this context of individualised optimization, where the idea of competition and responsibility appears, which demands that the new optimum behaviour are not, for the global system, less desirable than the previous one. However, usually the environment of the system, the global system, is discarded as a methodological principle.

Let us consider a system composed of two subsystems being a simple example of a hierarchically organised system. If both subsystems ignore the existence of the superior system that encompasses them and act in an effort to optimise their own processes, such systems are in competition because their coexistance and interrelation constitutes for both of them a constriction to their particular optimization. If the subsystems acknowledge the existance of the superior system and seek to optimize the state of the global system, these systems are in cooperation and their coexistence and interrelation constitutes for them a help. The competition, as it arises here, will tend to produce local optimizations which do not assure global optimization. The cooperation will tend to a global optimum, through relative optima, which will not necessarily coincide wit the absolute optimum of each subsystem.

2. Hierarchical concept of system

In this section, the ideas of the previous paragraph will be made more precise. We will base our discussion on a hierarchical definition of system (Graña, 1983) which supposes an extension of Definition 1.

Definition 4. Granted the existence of a set of irreducible elements, atomic elements, we shall define system, in a recursive way, as either

(a) a finite subset of the set of atomic elements together with the set of relations which link them; or

(b) a finite set of systems together with the set of relations which link them, where the relation between two systems at a non-elemental level will be given by the relations among components of each other.

We would like to point out three characteristics of this definition. The first is that the global system is considered as lacking environment. The reason for this is that we consider that every system of interest will be placed at an inner level of the hierarchy, for which its neighbourhood will be clearly defined. The second characteristic to notice is that the environment of the system under study is no longer considered as a set of isolated variables, but as a set of systems which conform to and are joined to, a system of superior order. The exogenous variables usually utilised in the models are, in fact, abstractions of these environment systems, and represent them totally or partially. Finally, we have to consider that this definition generalizes the concept of structure, from being only a concept of the inside of the system to the relationship among systems.

2.1. Control in hierarchical systems

A *process* is the transformation of the values associated with a set of variables. The process is the product of the system's activity, and it is a concept that makes Definition 2 and 3 of systems close together. The state variables are the variables upon which the process takes place, through which we observe the activity of the system. This conception applies to several disciplines, System Dynamics being among them.

We can express every process by a transformation law. System dynamics models are constructions of transformation laws according to a perspective derived from control theory. In a hierarchical structured system each of the systems, it is composed of, has its associated process and the transformation law that describes this. The elemental systems, the ones composed of atomic elements exclusively, will have a relatively simple associated process and transformation law. In contrast, systems at a higher level, composed of interrelated systems, will have a process and a transformation law being a composition, in general non-linear, of the ones correspond to the subsystems that comprise them [1]. In this way we

[1] Strictly speaking, the associated process of a system at a non-elemental level will be able to be described by multiple transformation laws which will capacitate the system for different outputs as a response to the same input, this means that each subsystem potentially is a set of subsystems being recognizable as belonging to the same class.

have extended the hierarchical concept. We have a hierarchy of processes and a hierarchy of transformation laws associated with the static structure of Definition 4.

Associated with every process exists a component of control whose activity consists in keeping the process unadulterated, that is to say, which makes the system accomplish its specific task. By component of control we mean a structure that can be external or internal to the system that carries out the process. In this latter case it can be implicit in the structure of the system or constituting a distinct subsystem. This component of control does not accomplish any optimizating activity, its regulatory activity takes part in the process of the system itself and is inherent to it.

Let us take from the control theory the following formulation of a system:

$$\dot{x} = f(x, u, t),$$

which represents the transformation law of a system, where $\mathbf{u}$ is the vector of control variables. We can classify these control variables in three groups, in such a way that

$$\mathbf{u} = [u_I, u_E, u_C],$$

where u_I are the control variables internally determined, u_E are those which come from the outside and u_C are the ones that remain constant throughout the whole life of the process of the system, the parameters of the system.

The system, including its internal control component, will now have the formulation:

$$\begin{aligned} &\dot{x} = f(x, u, t), \\ &u_E = \text{exogenous}, \\ &u_I = g(x, u, t), \qquad\qquad (1) \\ &u_C = \text{parameters}. \end{aligned}$$

The hierarchical conception of a system determines an associated hierarchy of control. In it the global system lacks an external control. If we consider a system at a certain level of the hierarchy, its exogenous variables are determined by the internal control of the system that includes it. The environment has become the control mechanism of the global system.

We can allocate the term *process freedom of a system* to a certain measure of the variety of possible behaviours of the system, or region of the state space accessible to the trajectories of the process, without modifying the structure of the system or its transformation law. If we take into account (1) we will notice that the only way to modify the behaviour of the system is through u_E (the initial conditions obviously do not enter into the discussion). It follows from this that the process freedom of a system without external control is nil. The overall system has no freedom, though the subsystems at different levels have, necessarily, a certain amount of process freedom. We can conclude also that every open system where u_E is not zero, will be inside a system which includes it.

2.2. Optimization in hierarchical systems

We can associate with every process or transformation law a performance index that evaluates the degree of perfection it is accomplished with. In the performance index are represented the results of the process and the criteria of optimization which describe the sense of the optimization and, ultimately, the optimum behaviour. This constitutes a measure of the evolution of the process, of the behaviour of the system.

The optimization is an activity acting upon the system, upon its structure or transformation law. The system, or external agents, can act upon the transformation law, changing it, in order that the performance index should reach an optimum value.

In a similar way to the existence of a control component, we can think of that of an optimization component, the *optimizer*. The optimizer can be partially internal to the system and constitutes a subsystem either separate from or implicit in the structure. The modification of the transformation law of the system, carried out by the optimizer, suggests a concept of freedom wider than the process freedom, *the structural freedom*. This is a measure of the set of transformation laws that can be associated with a system. Taking again the control theory notation [2], at the system (1) we observe that the only way to modify the transformation law of the system is to modify $\mathbf{u}$. The structural freedom includes the process freedom, in the sense that every modification of $\mathbf{u}$ is a

[2] Such a notation is totally unable to represent the existence of an optimizer, nevertheless we fall back on it to illustrate our exposition. Mesarovic's theory of hierarchical systems (Mesarovic, 1970) provides a formal approach to some of the ideas of this paper.

modification of the structure $x = f(x, u, t)$. The structural freedom includes also a freedom of design of u_I and $u_I = g(x, u, t)$. With respect to the optimizer, the structural freedom determines the constrictions to optimization implicit in the structure of the system itself and the way in which the optimizer can modify it.

The tasks of the optimizer can be stated as:
- Creation of the optimization criteria.
- Construction of the performance index.
- Inference of the optimal control policies.
- Implementation of the policies.

These tasks can be assumed to a greater or lesser degree by the system itself. The greater the proportion of the internal optimization is, the greater the freedom of design must be, as it is the only internal structural freedom. Therefore to interiorise the optimization entails a greater complexity of the system.

We can outline a taxonomy of systems according to the degree of optimization functions which are internal to the system:

- Systems without any internal optimization function. In this kind of system u_E determines the process, for instance, tools.

- Systems able to carry out the policies externally determined. In these systems u_E is the externally deduced policy. To this category belong elementary automatic systems.

- Systems able to deduce and implement policies internally. These are more complex automatic systems.

- Systems able to construct performance indices from some externally fixed criteria. These are systems with a certain logic capacity.

- Systems able to create criteria. These are systems that can decide their purpose. In this kind of systems u_E has the character of information needed to create the criteria.

In the overall system all the optimization functions are internal ones and the systems at different levels will belong to any of the mentioned categories. If more than one system belongs to the latter, the idea of partial optimization arises and it is in this context that the behaviour of competition or cooperation must be seen. The appearance of one or the other will depend on the particular characteristics of the optimization criteria which lead to the formulation of the performance index in each subsystem. If the performance index does not take into account the existence of a superior order system, any optimization strategy of the system will be a strategy of competition. If, on the contrary, the performance indices of the systems acknowledge the existence of an overall system any optimization strategy will be a strategy of cooperation. In any case, there must exist a relation between the criteria of the overall system and the criteria being defined by the subsystems. The overall system can influence the creation of the criteria of the subsystem through the establishment of the external control components u_E on each subsystem, namely, the relations among subsystems.

3. Hierarchical modelling

The hierarchical structure is that most used by man when working with very complex systems. It is a fact that many spontaneous social organizations, built without a previous structural design, have this kind of structure and it also appears in many of our mental processes.

There are two general modelling methods. The first, the 'iterative' or 'successive method, proposes an initial model and transforms it until it fits the reality we want to form a model of, or it shows the ideal we want to elaborate. To transform each model we may add or delete elements and we may add, delete or modify the relationships among them. The initial model is approximated by stages to the desired model and modelling is an iterative process of conceptualization–formulation–verification.

On the other hand, in hierarchical modelling we build a set of models of progressively decreasing abstraction until arriving at the model with the desired abstraction level. We mean by abstraction of a model the degree of detail with which it is constructed. Hierarchical modelling assumes the existence of a hierarchical structure being the essential structure of the system. The chain of models of progressively decreasing abstraction levels is the set of models that describes the system in its different levels of organization. The relation among models in the chain is of 'explanation'. Each model explains the previous one with more detail and the previous model is implicit in the present model. In this way, if we observe simultaneously the chain of models, we obtain the system description as a hierarchy.

Taking the iterative method as a guide for modelling, the assumed abstraction level must be the lower, the level of the definitive model. The complexity of the systems increases as the degree of detail increases, so the construction of the model will be necessarily very complex. A strategy to elude this complexity is to reverse the hierarchical method. We focus the construction on a subsystem and we approach the definitive model by successive boundary extensions. In our opinion this view of the modelling process obscures the general structure of the system whilst the hierarchical method makes it evident from the beginning, without obscuring the microstructure.

3.1. Aggregation and abstraction

Aggregation is a concept in common use in System Dynamics as well as in other modelling techniques. Its meaning is related to the degree of detail of the description, but it is a particular case, the simplest, of the broader concept of abstraction.

An aggregate variable supposes the existence of a group of variables that represent objects of the same class which can be subsumed in the aggregated variable. The aggregation consists in a lineal composition of variables that have a high degree of homogeneity and belong to a well-defined class. On the other hand, the abstraction consists in the composition of variables, not necessarily homogenous, that can produce a system qualitatively different from its components.

We can consider aggregation to be a particular case, albeit an important one, of abstraction, in which the composition is lineal and the objects homogenous. The levels of aggregation are also levels of abstraction and their use in modelling produce an impression of hierarchical modelling. However, we want to point out that the transformation from a level of aggregation to a lower one is relatively simple, because the decomposition of a variable is, in practice, the enunciation of the elements having the same character. On the contrary, the descent from one abstraction level to a lower one requires the construction of a model for each decomposed element and the synthesis of all these models into a compact entity.

3.2. Hierarchical modelling in system dynamics

System dynamics flow diagrams suppose a representation of the system in a static way. The level variables correspond to the system components and the rates and auxiliaries are quantifiers of the relations among components, and from the perspective of the flow diagram they serve to define these relations. So the flow diagram is the tool used in system dynamics for the conceptualization of models according to he view of the first definition of system. The resulting graphs of the simulation represent the system according to the second definition. The DYNAMO equations or the differential equations specify the system representation according to the third definition. To introduce hierarchical modelling into the system dynamics practiced we shall propose a hierarchical definition of model using the elements of system dynamics. This definition is a static definition, so it preserves the previously stated correspondence among levels, rates and auxiliaries of the flow diagram and components and relations of the static definition of system.

Definition 5. A model of a system is either

(a) a finite set of level variables and their relations, determined by the linkage of rates and information, forming feedback loops; or

(b) a finite set of models and their relations, determined by the information nets and the flows among models forming feedback loops. The information and flow relations among models will be determined by the relations among components of these models.

Following the hierarchical method of modelling, at a certain abstraction level we will find that the system components are systems in themselves. To be capable of making models of an abstraction level higher than the elemental one, we must consider that the level variables of the flow diagram can represent not only atomic components but components which are systems. This generalization allows us to make, at each abstraction level, models which can be formulated, simulated and validated and so we can verify at each step our hypothesis. As a final result, we have a set of models, each one with all the characteristics of the system dynamics models, which describe the system at various levels of organization or detail.

The way a model of lower abstraction level is built from the previous, more abstract, one is not evident in the most general case. We have said that the level variables of the flow diagram can represent systems, but abstraction is not only inherent to the level variables. The rate variables and the information net which affects a level variable can be themselves abstractions of a set of rates and auxiliaries belonging to a lower abstraction level. Therefore the translation of a model to the following abstraction level involves all kinds of variables and we do not know the way to formalize this transition. Experimentatation with this philosophy of modelling will show, among other things, the way to do it.

Randers (1980) points out that the best way to define the problem under study is to focus on its process character, establishing a reference mode. The application of this idea to the hierarchical method of modelling brings us to a hierarchical set of reference modes. Each one will be a guide in the construction of a model at a given abstraction level. However, information needed to built up these reference modes reflects in itself a real structuration which may not fit the structuration we want to create. Therefore, available information can allow the construction of reference modes at certain organizational levels, probably the highest ones, but not at all levels.

Finally, considering the models as theories, expressions in a formal language of the knowledge about a system, hierarchical modelling procures for us a set of models/theories of decreasing generality. The models at the highest abstraction levels are theories of great generality capable of being transplanted from one system to another of a different nature, belonging to other areas of knowledge.

4. Conclusions

The hierarchical conception of system gives a structure to their environment, enhancing their systemic nature and the need for its interaction through the boundary of the system. The environment is responsible for the control of a system process. If all the control of a system were internal to it the system would not have any process freedom. The definition of the purpose of a system, along with its optimization, is a shared responsibility between the system and its environment. Attending to the number of optimization functions which are internal to the system, we can define a taxonomy of systems. Interactions of competition and cooperation appear in the systems which are capable of creating criteria for the definition of the performance index. If in the performance index the overall system is implicit, the strategy will be one of cooperation. If not, the strategy will be of competition.

System dynamics, as a methodology of modelling, can take in the hierarchical conception of system and use it as a guide for the construction of a set of progressively decreasing abstraction models. The praxis of modelling may not turn out to be very different from that which is habitually used, but conceptually seems to clarify the structure of the system better than the usual conception, which progressively spreads the boundary in a horizontal scope.

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