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Lattice Associative Memories for Segmenting Color Images in Different Color Spaces

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Overview

1. Introduction
2. Background
3. Segmentation with the *WM* method
4. Segmentation results in different color spaces
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1. Introduction

Segmentation

- A preliminary step in the description and representation of regions of interest in an image.

- To divide an image X into a finite set of disjoint regions $\mathcal{R}_i$ whose pixels share well defined attributes (hue, intensity, texture).

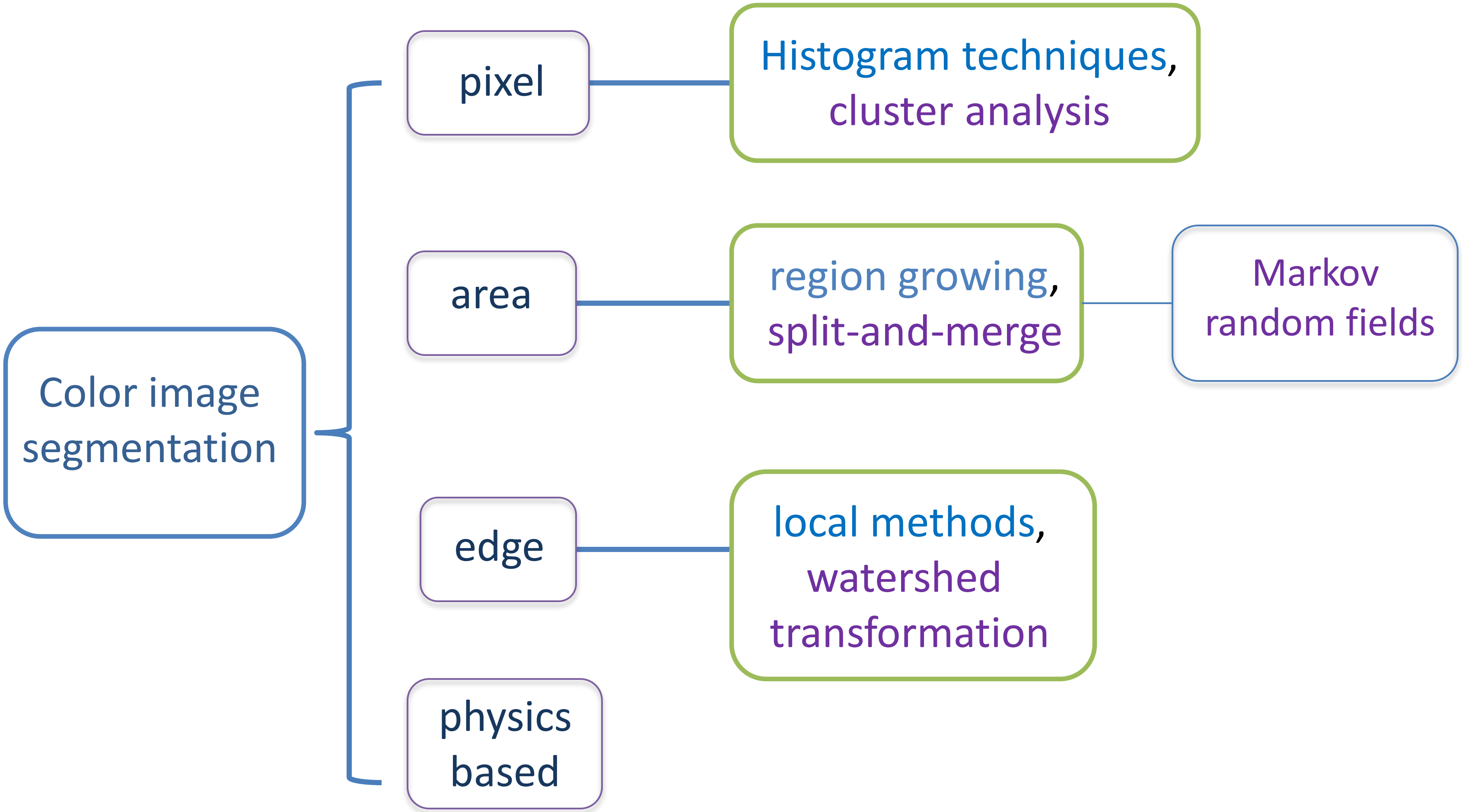
$$1)\ \bigcup_{i=1}^{q} \mathcal{R}_i = X$$

$$2)\ \mathcal{R}_i \cap \mathcal{R}_j = \varnothing \quad \text{for } i \neq j$$

$$3)\ p(\mathcal{R}_i) = \mathbf{true},\ \forall i$$

$$4)\ p(\mathcal{R}_i \cup \mathcal{R}_j) = \mathbf{false} \quad \text{for } i \neq j$$

1. Introduction

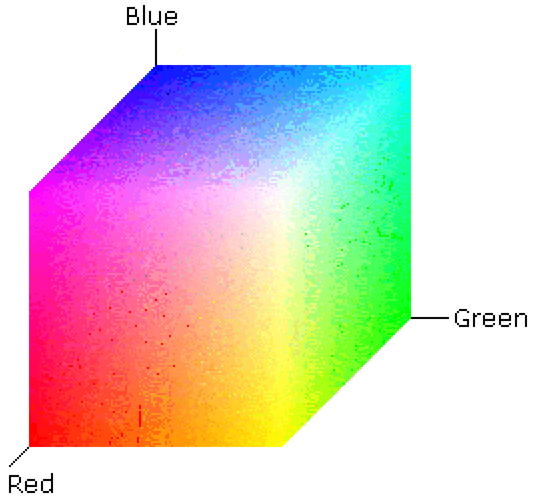


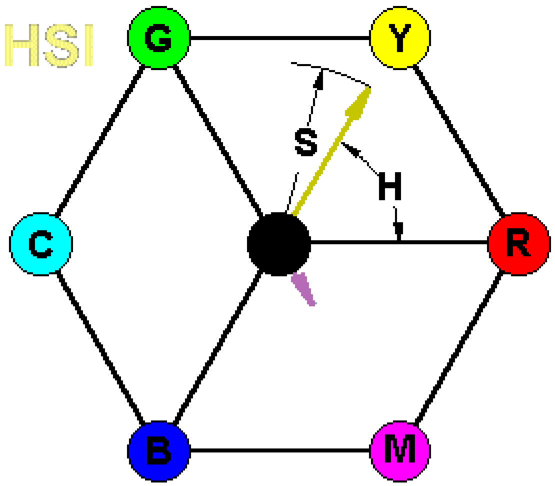
1. Introduction

- We describe a lattice algebra based technique for image segmentation applied to RGB color images transformed to other representative systems.

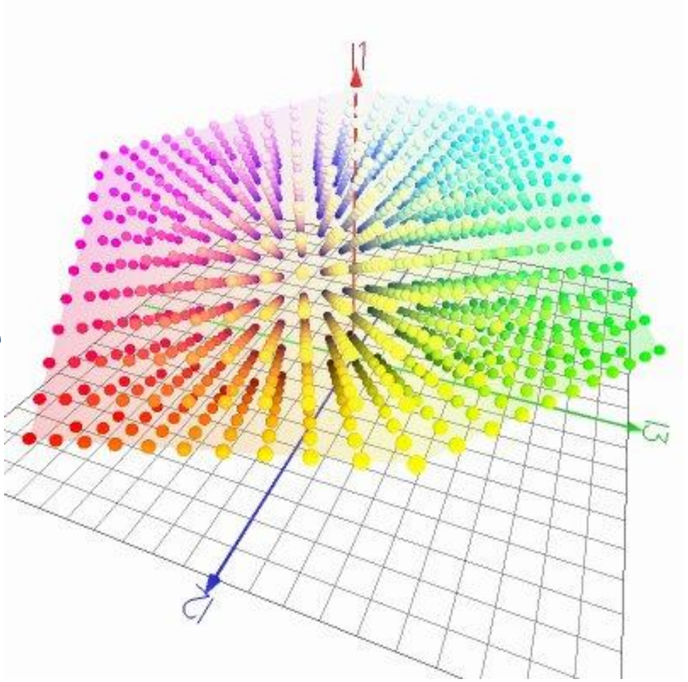
HSI (*hue, saturation, intensity*)

Humans describe a color object by its hue, saturation and intensity.





1. Introduction



I1I2I3 (*principal components approximation*)

Linear decorrelation of the RGB color space

I1I2I3 are orthogonal color components

L*a*b* (*luminance, redness/greeness, yellowness/blueness*)

It describes all the colors visible to the human eye, created to be used as a reference

Perceptually uniform: a change of the same amount in a color value should produce a change of about the same visual importance

1. Introduction

- The method relies on the *min*-W_{xx} and *max*-M_{xx} lattice auto-associative memories, with X being the set formed by all different colors or 3-D pixel vectors.

- The scaled column vectors of either memory together with the *minimum* and *maximum* vector bounds of X may form the vertices of tetrahedra enclosing subsets of X (most satured colors).

2. Background

2.1 Lattice algebra operations

We define the lattice operations for all $i = 1, \ldots, m$ and $j = 1, \ldots, n$, as follows:

1. $(X \vee Y)_{ij} = x_{ij} \vee y_{ij}$ maximum
2. $(X \wedge Y)_{ij} = x_{ij} \wedge y_{ij}$ minimum
3. $X \leq Y \Leftrightarrow x_{ij} \leq y_{ij}$ inequalities
4. $X^* = -X^t = -x_{ji}$ conjugate matrix
5. $(X \boxed{\vee} Y)_{ij} = \bigvee_{k=1}^{p}(x_{ik} + y_{kj})$ *max-sum*
6. $(X \boxed{\wedge} Y)_{ij} = \bigwedge_{k=1}^{p}(x_{ik} + y_{kj})$ *min-sum*
7. $\mathbf{y} \times \mathbf{x}^t = \mathbf{y} \boxed{\vee} \mathbf{x}^t = \mathbf{y} \boxed{\wedge} \mathbf{x}^t = (y_i + x_j)$ *outer sum.*

2. Background

2.2 Lattice associative memories (*LAAMs*)

Given a set of vectors $(\mathbf{x}^\xi, \mathbf{y}^\xi) \in \mathbb{R}^m \times \mathbb{R}^n$ for $\xi = 1, \ldots, k$ associated in a pair of matrices (X, Y), where $X = (\mathbf{x}^1, \ldots, \mathbf{x}^k) \in$ and $Y = (\mathbf{y}^1, \ldots, \mathbf{y}^k)$. The min-*memory* W_{XY} and the max-*memory* M_{XY}, both of size $m \times n$, that store a set of associations (X, Y) are given, respectively,

$$W_{XY} = \bigwedge_{\xi=1}^{k} [\mathbf{y}^\xi \times (-\mathbf{x}^\xi)^t]\,; \;\; w_{ij} = \bigwedge_{\xi=1}^{k} (y_i^\xi - x_j^\xi), \tag{1}$$

$$M_{XY} = \bigvee_{\xi=1}^{k} [\mathbf{y}^\xi \times (-\mathbf{x}^\xi)^t]\,; \;\; m_{ij} = \bigvee_{\xi=1}^{k} (y_i^\xi - x_j^\xi). \tag{2}$$

We speak of a *lattice hetero-associative memory* if $X \neq Y$.

For the case $X = Y$ we have a *lattice auto-associative memory*.

2. Background

2.2 Lattice associative memories (*LAAMs*)

- LAAMs are non linear memories introduced as a new paradigm to deal with the problem of recalling exemplar patterns from noisy of real valued inputs [1].

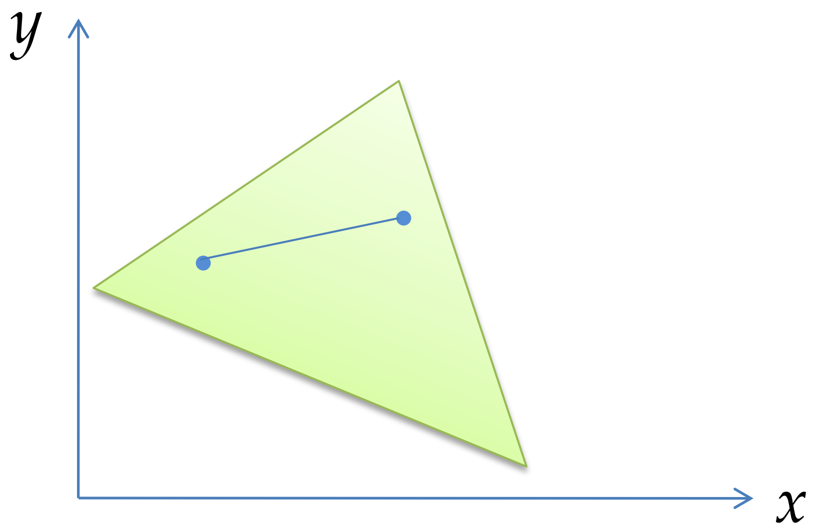
- Recent applications include the determination of endmembers in hyperspectral images [2].

[1] Ritter, Sussner, Diaz de León, (1998), [2] Ritter, Urcid, Schmalz, (2008).

3. Segmentation with the *WM* method

3.1 Preliminaries

- The method is based on the geometry of convex sets.

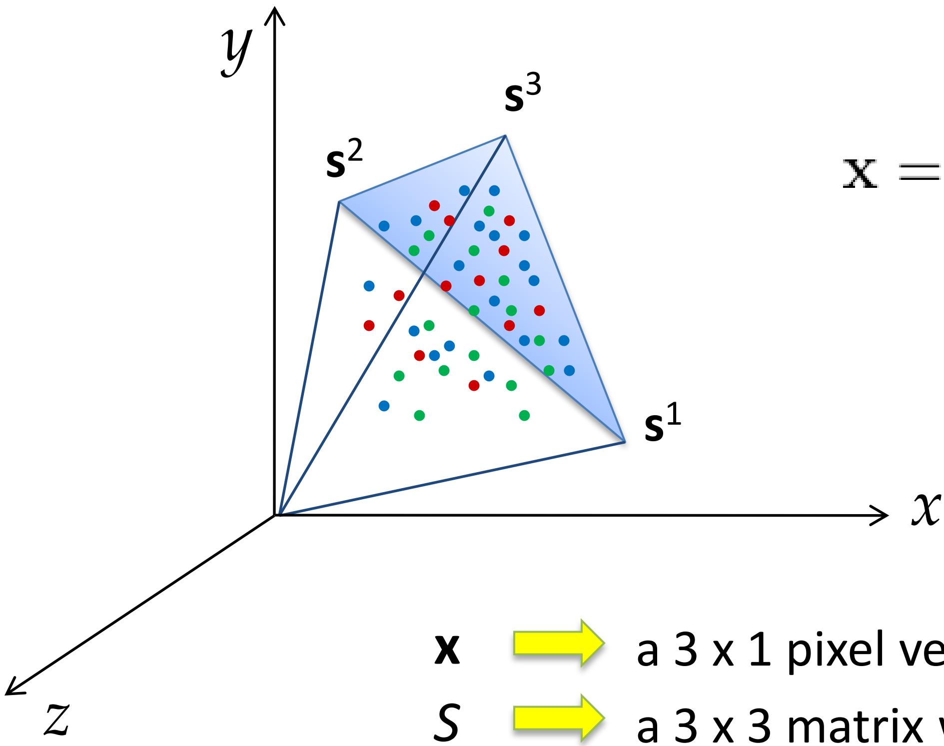


- The minimal convex set formed by $n + 1$ vertices is the simplex.
- Since the color solid is a subspace of R^3, a 3-dimensional simplex will be a tetrahedron.

3. Segmentation with the *WM* method

3.2 The linear mixing model

- Considering pixel vectors in a color image as enclosed by some tetrahedron:



$$\mathbf{x} = S\psi = \psi_1 \mathbf{s}^1 + \psi_2 \mathbf{s}^2 + \psi_3 \mathbf{s}^3$$

$$\psi_1 + \psi_2 + \psi_3 = 1$$

$$\psi_1, \psi_2, \psi_3 \geq 0 \quad (3)$$

x → a 3 x 1 pixel vector,

S → a 3 x 3 matrix with the most satured colors

ψ → a 3 x 1 vector of color fractions present in x.

3. Segmentation with the *WM* method

3.3 Procedure

1. Given a color image *A* of size *p* x *q*, the *X* set containing all different colors (3D vectors) present in *A* is formed.

2. The memory matrices *min*-W_{XX} and *max*-M_{XX} are computed according to eqs. (1) and (2).

3. Since the vector columns of $W = (\mathbf{w}^1, \mathbf{w}^2, \mathbf{w}^3)$ and $M = (\mathbf{m}^1, \mathbf{m}^2, \mathbf{m}^3)$ may not belong to the numerical range of a given color space, an additive scaling is required.

3. Segmentation with the *WM* method

3.3 Procedure

The *minimum* and *maximum* vector bounds of $X = (\mathbf{x}^1, \ldots, \mathbf{x}^k)$ are given by $\mathbf{v} = \wedge_{\xi=1}^{k} \mathbf{x}^{\xi}$ and $\mathbf{u} = \vee_{\xi=1}^{k} \mathbf{x}^{\xi}$, respectively. Then additive scaling results in two scaled matrices denoted $\overline{W}$ and $\overline{M}$, whose column vectors are defined by

$$\overline{\mathbf{w}}^i = \mathbf{w}^i + u_i = \mathbf{w}^i + \bigvee_{\xi=1}^{k} x_i^{\xi} \quad ; \quad \overline{\mathbf{m}}^i = \mathbf{m}^i + v_i = \mathbf{m}^i + \bigwedge_{\xi=1}^{k} x_i^{\xi}$$

Each set $\{\overline{\mathbf{w}}^1, \overline{\mathbf{w}}^2, \overline{\mathbf{w}}^3\}$ or $\{\overline{\mathbf{m}}^1, \overline{\mathbf{m}}^2, \overline{\mathbf{m}}^3\}$ makes possible to determine several tetrahedra enclosing specific subsets of *X*.

3. Segmentation with the *WM* method

4. The present step consists of solving eq. (3) making $S = \overline{W}$ or $S = \overline{M}$ to find vector ψ for each $\mathbf{x} \in X$ (NNLS).

5. Once we have found vectors ψ for every pixel in the image, these are reassembled into grayscale fraction images for $\mathbf{s}^1$, $\mathbf{s}^2$, $\mathbf{s}^3$.

3. Segmentation with the *WM* method

Example

Figure 1. RGB color image of size 128 x 128 and its HSI transformation displayed as a false color image.

3. Segmentation with the WM method

The set $X = \{\mathbf{x}^1, \ldots, \mathbf{x}^{13{,}844}\}$ from a total of 16,384 pixel vectors.

$$\overline{W} = \begin{pmatrix} 255 & 100 & 36 \\ 188 & 255 & 16 \\ 115 & 103 & 255 \end{pmatrix} \quad , \quad \mathbf{u} = \begin{pmatrix} 255 \\ 255 \\ 255 \end{pmatrix}$$

$$\overline{M} = \begin{pmatrix} 0 & 67 & 140 \\ 155 & 0 & 152 \\ 219 & 239 & 0 \end{pmatrix} \quad , \quad \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3. Segmentation with the *WM* method

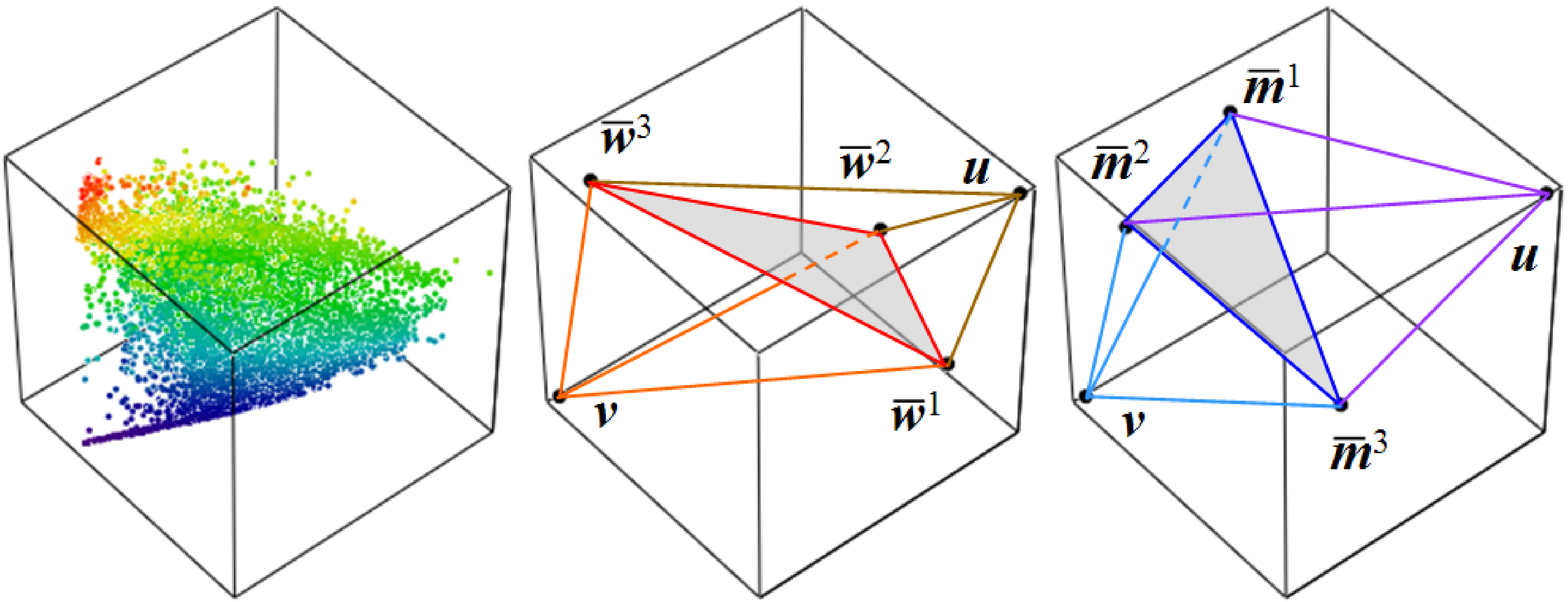


Figure 2. Left: 3D scatter plot of *X* showing all different colors in the HSI representation of "peppers" image; right: tetrahedra determined from $\overline{W}$ and $\overline{M}$.

3. Segmentation with the WM method

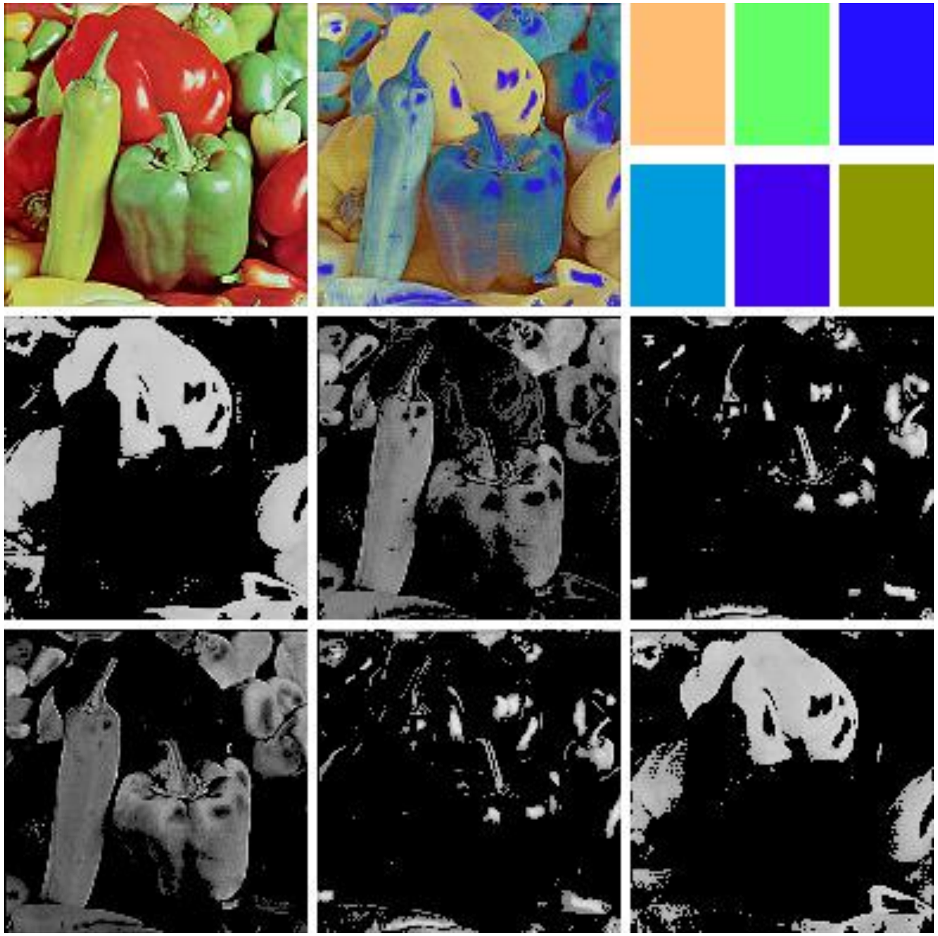


Figure 3. 1st row: RGB color image, transformed HSI color image, satured colors obtained from the scaled columns of W and M; 2nd and 3rd rows: grayscale segmented images derived from $\overline{W}$ and $\overline{M}$.

4. Segmentation results in other color spaces

- The performance of the *WM* method was tested in representative color spaces: RGB, I1I2I3, HSI and L*a*b*.

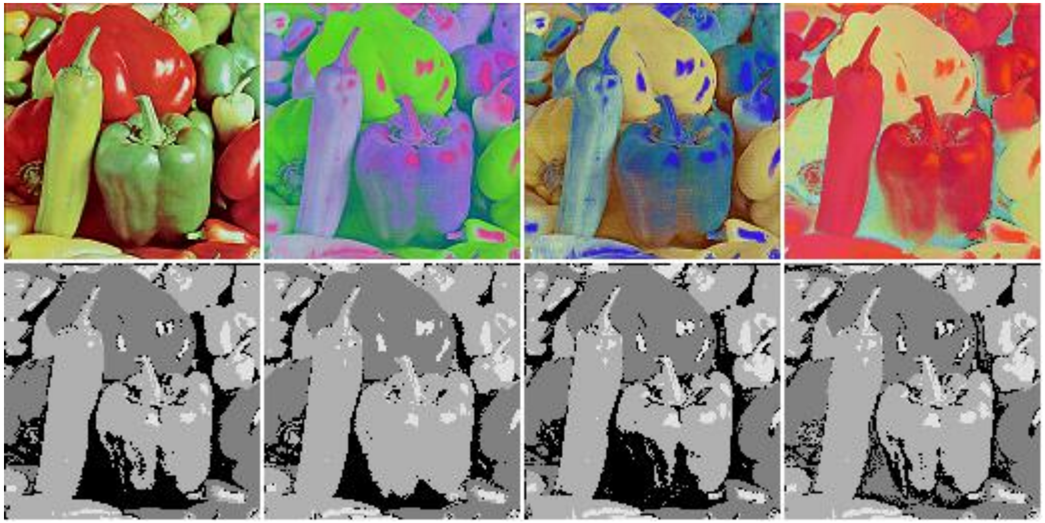


Figure 4. 1st row: RGB color image and its transformation to I1I2I3, HSI and L*a*b* color spaces; 2nd row: composed thresholded fraction maps selected from $\overline{W}$ and $\overline{M}$.

4. Segmentation results in other color spaces

Table 1. Color fractions of associated scaled columns selected to form the composite images in figure 4.

Color Space	Selected Columns
RGB	w^1, w^2, m^1
$I_1I_2I_3$	w^1, w^3, m^3
HSI	w^1, w^3, m^1
L*a*b*	w^1, w^2, m^2

4. Segmentation results in other color spaces

Comparison of results

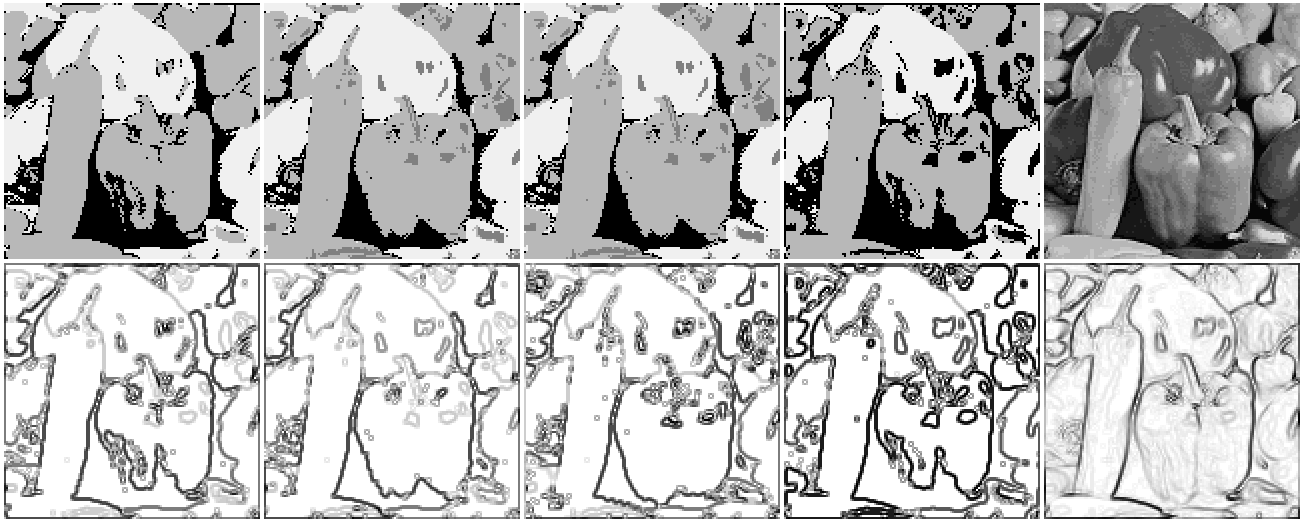


Figure 5. 1st row: segmented images produced by the RGB and I1I2I3 color spaces, a Mahalanobis distance clustering method, and an hybrid technique employing histrograms and morphological watersheds; 2nd row: Sobel gradient edge images obtained from the previous images.

4. Segmentation results in other color spaces

Comparison of results

Table 2. Segmentation performance for the “peppers” color image.

Segmentation Method	Corr. Coef.	SNR
WM in RGB	0.707	14.179
WM in I1I2I3	0.717	14.931
WM in HSI	0.708	14.124
WM in L*a*b*	0.675	14.006
Mahalanobis distance clustering	0.632	12.912
Histograms + Morph. Watersheds	0.594	9.814

4. Segmentation results in other color spaces

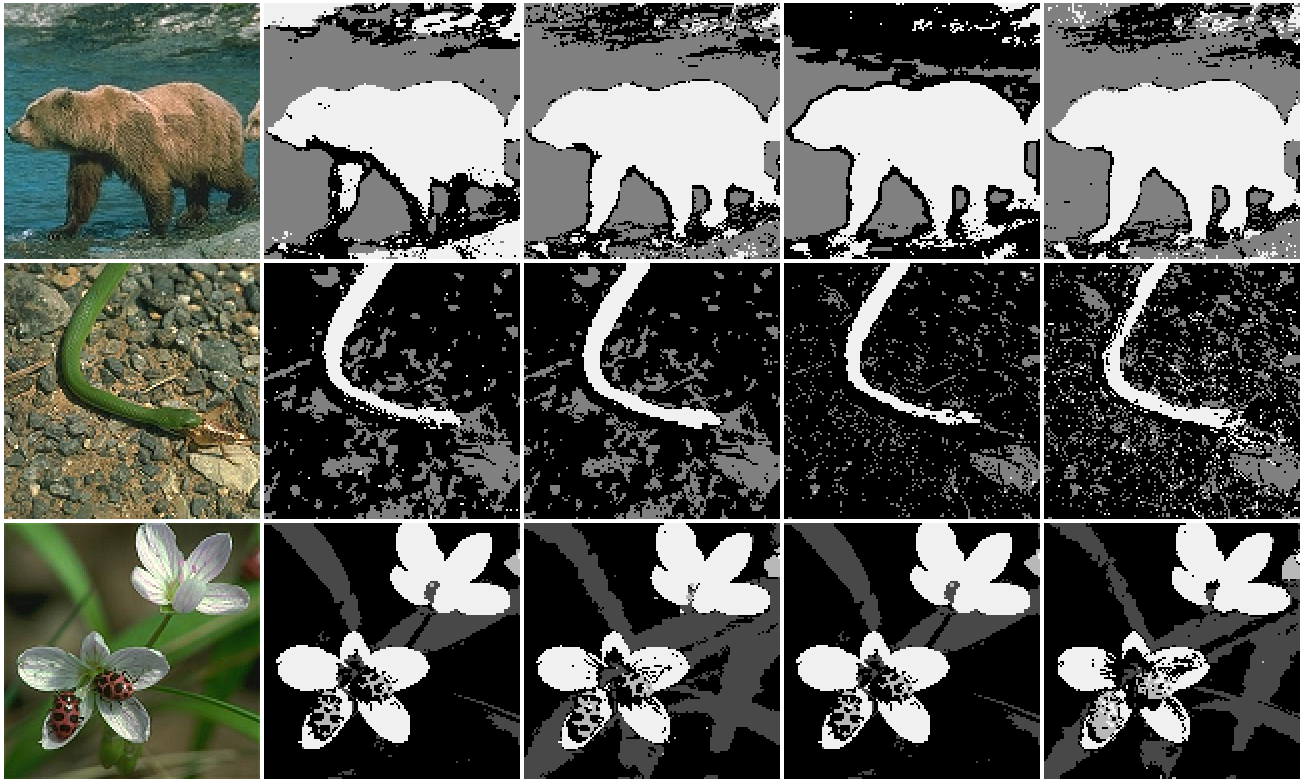


Figure 6. 1st column: sample RGB color images; 2nd to 5th columns: compound segmented images obtained with the *WM* method, respectively, in the RGB, I1I2I3, HSI and L*a*b* color spaces, main regions of interest are quantized .

5. Conclusions

- This work describes a color image segmentation method in different color spaces based on LAAMs.

- The scaled column vectors of W and M define the most satured pixels or extreme points.

- The extreme points are suitable to perform semi-constrained linear unmixing to describe color fractions at any other pixel.

5. Conclusions

- Some examples are given to illustrate the results and a preliminary comparison against two other segmentation techniques.
- We remark that the LAAMs based approach can be classified as an efficient unsupervised pixel clustering technique.

Thanks for your attention !

any questions ?