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## Median Hetero-Associative Memories Applied to the Categorization of True-Color Patterns



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# ORGANIZATION OF THE PRESENTATION

1. Introduction.
2. Foundations about Associative Memories.
3. Short state of the art on Associative Memories.
4. Basics on Median Associative Memories.
5. Present a first study of the behavior of MED-AMs.
6. An Application: Image Categorization using MED-HAMs.
7. Conclusions and present work.

# INTRODUCTION

The concept of associative memory (AM) emerges from psychological theories of human and animals learn.

A memory stores information by learning correlations among different stimuli.



When a stimulus is presented as a memory cue, the other is retrieved as a consequence; the two stimuli have become associated each other in the memory.

# INTRODUCTION

The concept of associative memory (AM) emerges from psychological theories of human and animals learn.

A memory stores information by learning correlations among different stimuli.



When a stimulus is presented as a memory cue, the other is retrieved as a consequence; the two stimuli have become associated each other in the memory.

# INTRODUCTION

An AM, What is it?

An AM, is a **device useful** to associate patterns or concepts or objects.

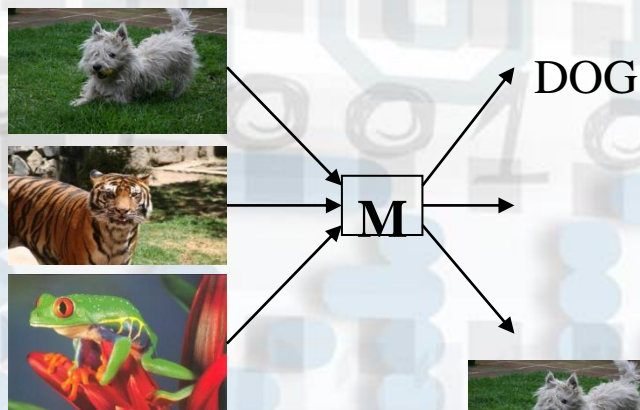
An Am is an input-output device that associates input patterns (**keys**) and output patterns (**patterns to be recalled or reconstructed**):



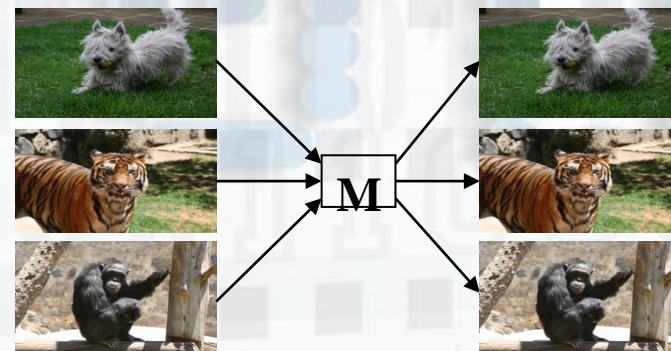
# INTRODUCTION

There are three main classes of AMs:

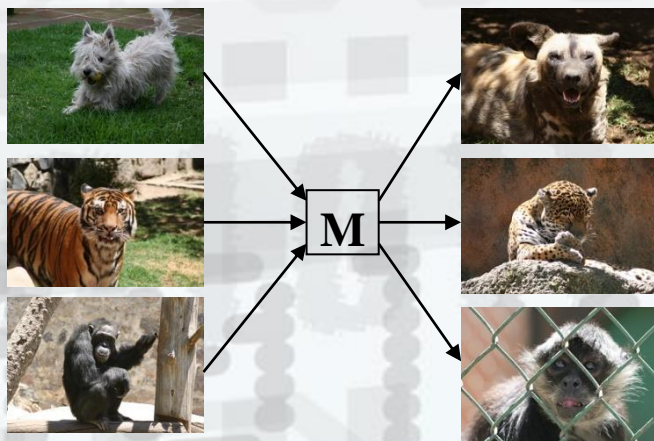
Hetero-associative:



Auto-associative:



Bi-directional:



# FOUNDATIONS

A pattern is represented as a vector:



Input or key  
vector:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Output  
vector:

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$

# FOUNDATIONS

Each pattern  $\mathbf{x}$  forms an **association** with a corresponding output pattern  $\mathbf{y}$ .

An **association** between input pattern  $\mathbf{x}$  and output pattern  $\mathbf{y}$  is denoted as  $(\mathbf{x}, \mathbf{y})$ .

For  $k$  integer and positive, the corresponding association will be denoted as  $(\mathbf{x}^k, \mathbf{y}^k)$ .

The associative memory  $\mathbf{M}$  is represented by a matrix whose  $ij$ -th component is  $m_{ij}$ .

$\mathbf{M}$  is generated from a finite a priori set of known associations, known as the **fundamental set of associations**, or simply the **fundamental set** (FS).



# FOUNDATIONS

If  $k$  is an index, the fundamental set (FS) is represented as:

$$\left\{ \left( x^k, y^k \right) \mid k = 1, 2, \dots, p \right\}$$

with  $p$  the cardinality of the set.

The patterns that form the fundamental set are called *fundamental patterns*.

If it holds that  $x^k = y^k \quad \forall k \in \{1, 2, \dots, p\}$   $\mathbf{M}$  is auto-associative.

otherwise it is hetero-associative.

# FOUNDATIONS

A distorted version of a pattern  $\mathbf{x}$  to be recalled will be denoted as:  $\tilde{\mathbf{x}}$

If when feeding a distorted version of  $\mathbf{X}^k$  with  $w \in \{1, 2, \dots, p\}$

to an associative memory  $\mathbf{M}$ , it happens that the output corresponds exactly to the associated pattern  $\mathbf{y}^k$ , we say that recalling is perfect.

If this hold for all  $k$ ,  $\mathbf{M}$  has perfect recall.

# FOUNDATIONS

Learning:

Input pattern:

$x_1^k$   $x_2^k$   $\dots$   $x_j^k$   $\dots$   $x_n^k$

Output pattern:

$y_1^k$   
 $y_2^k$   
 $\vdots$   
 $y_i^k$   
 $\vdots$   
 $y_m^k$

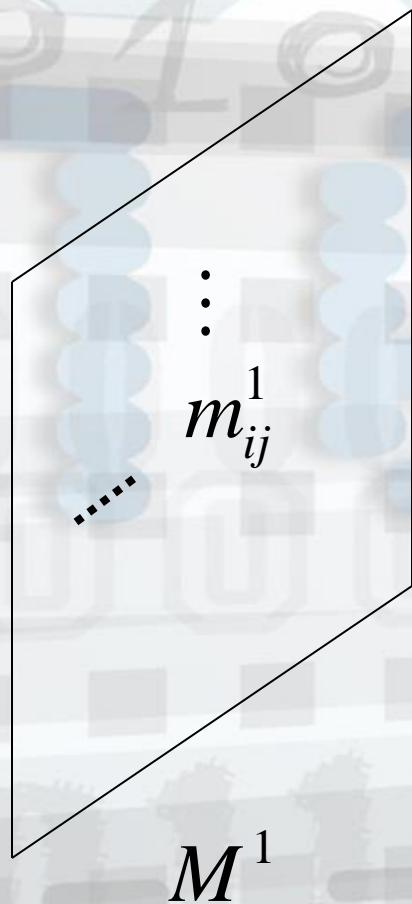
		$\vdots$	
	$\dots$	$m_{ij}^k$	$\dots$
		$\vdots$	

**The problem consists on computing the weights**

$M^k$

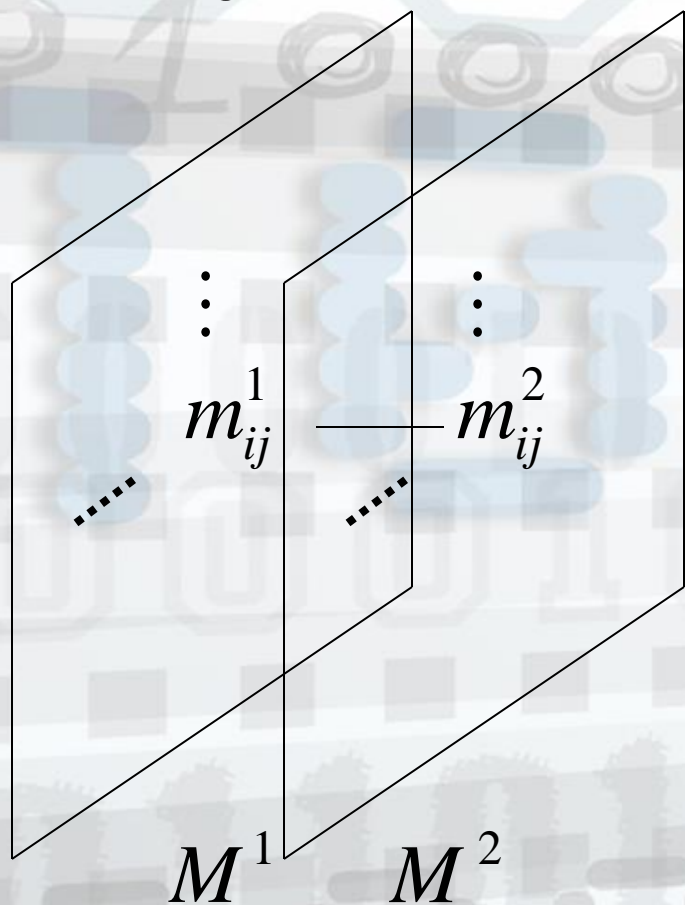
# FOUNDATIONS

Learning (we do this for each association):



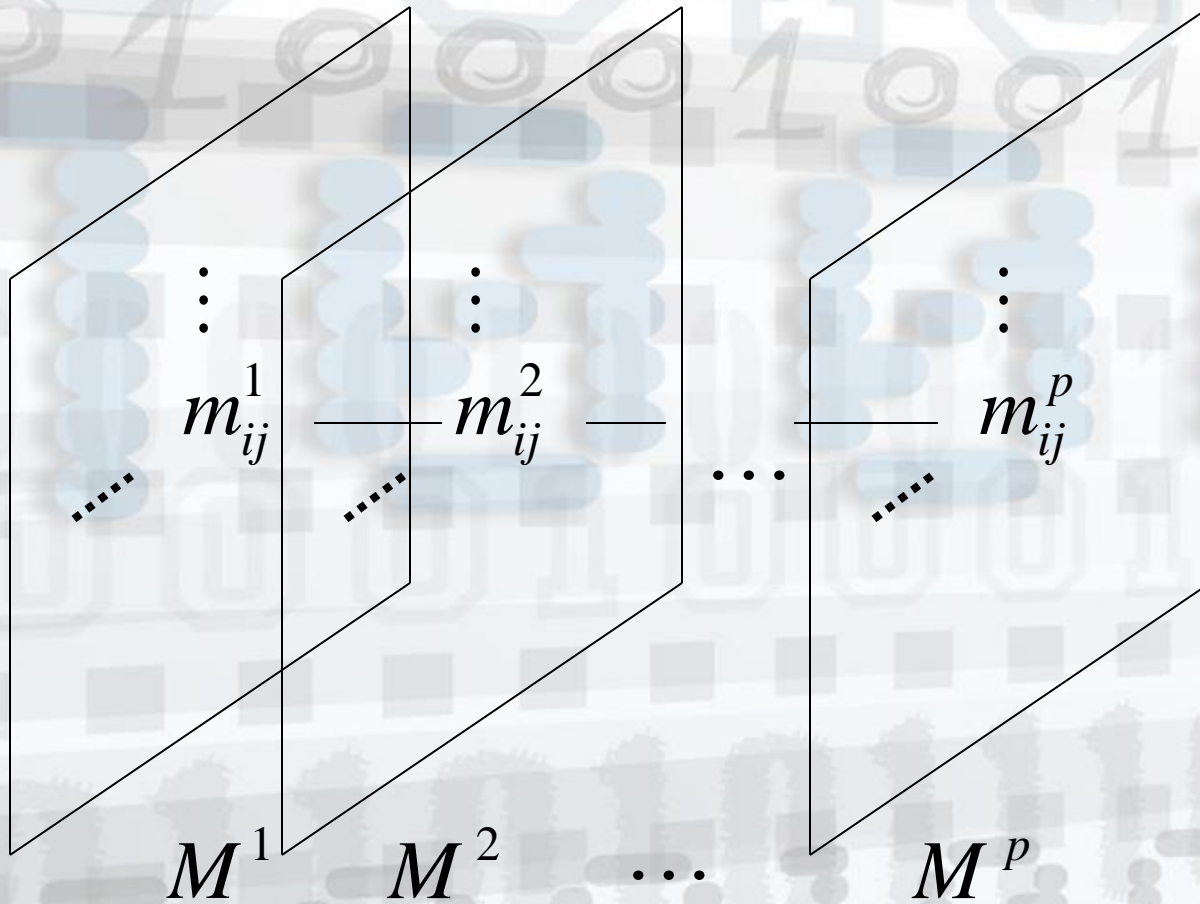
# FOUNDATIONS

Learning (we do this for each association):



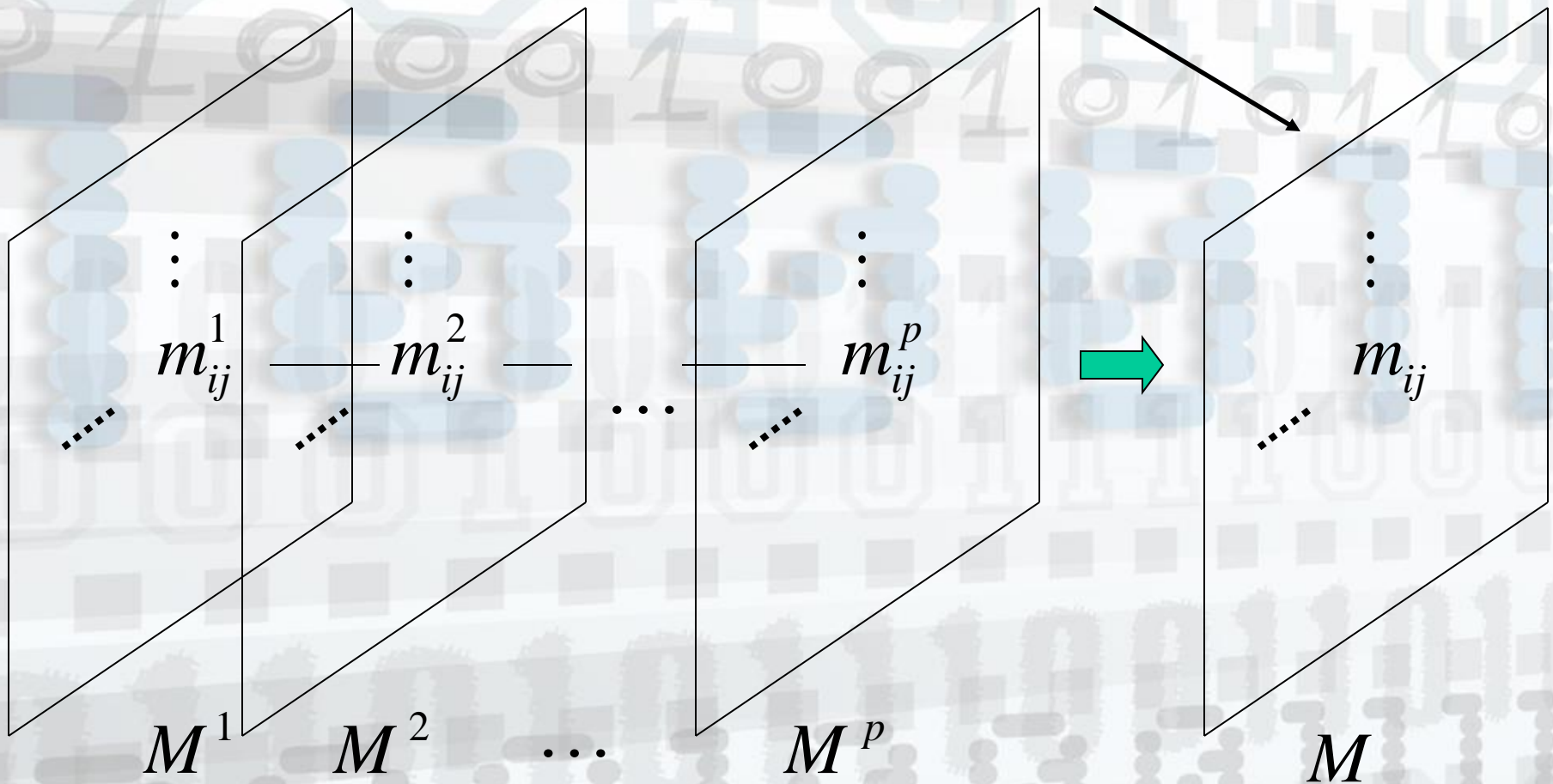
# FOUNDATIONS

Learning (for each association a partial memory is built):



# FOUNDATIONS

Learning (from these partial matrices a **global matrix** is built):



This is done with a combination of internal operators and external operators.

# FOUNDATIONS

In the case, for example, of Morphological Associative Memories:

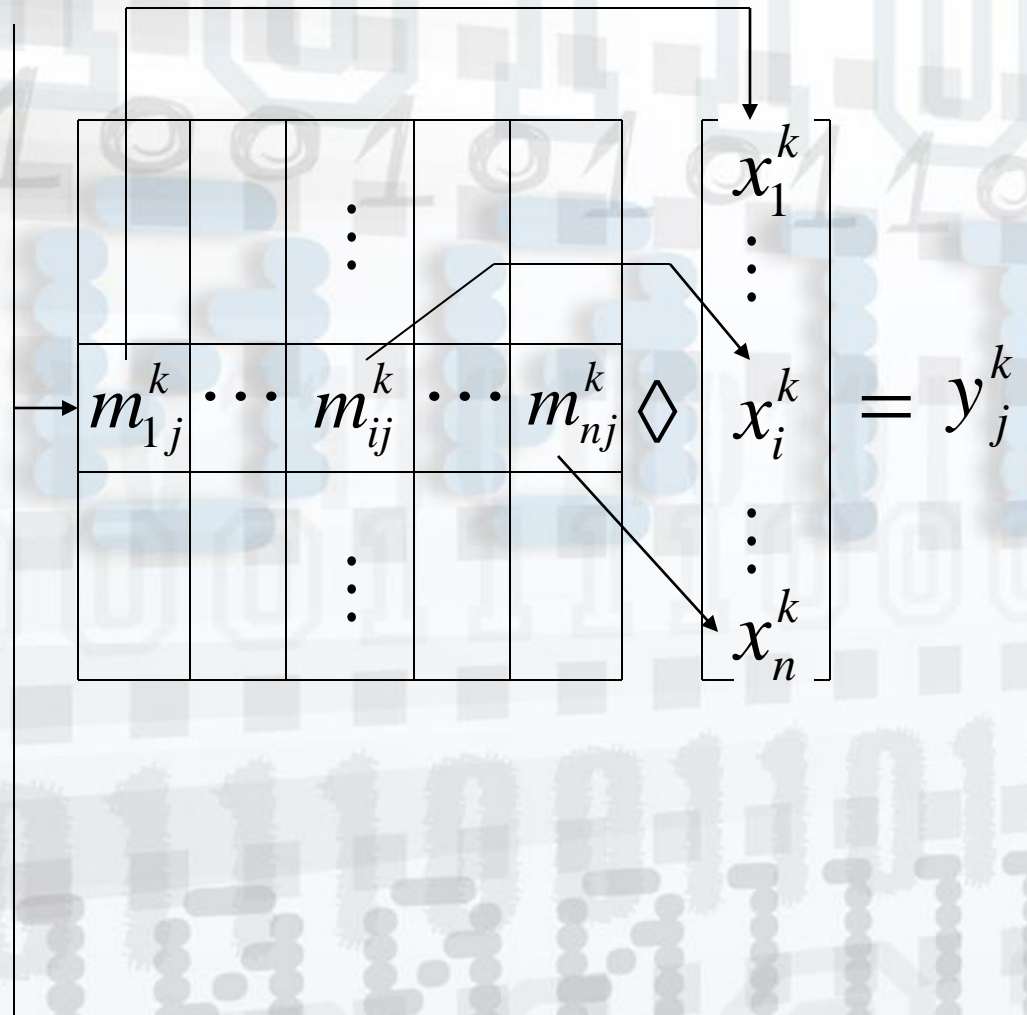
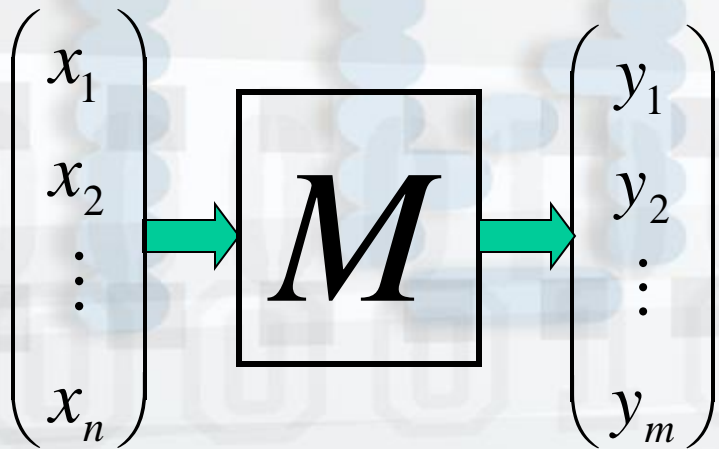
For learning, internal operation is a **subtraction** and the external operation is either a **min** or a **max**.

For retrieval, internal operation is an **addition** and the external operation is either an **max** or a **min**.



# FOUNDATIONS

Pattern recall:



## SHORT STATE OF THE ART ON AMS:

For a complete list of related papers refer to the reference section of the paper or:

**R. A. Vázquez and H. Sossa (2010). A Comprehensive Survey on Associative Memories. Submitted to Neurocomputing.**

# GOAL OF THE INVESTIGATION:

Recently in:

**R. A. Vázquez and H. Sossa (2009). Behavior of Morphological Associative Memories with True-Color Image Patterns. Neurocomputing 73(1-3):225-244.**

In this paper:

1. We investigate the behavior of Median Associative Memories (MED-AMs).
2. We present their application in the problem of categorization of images.

# BASICS ON MEDIAN ASSOCIATIVE MEMS:

Median associative memories (MED-AMs) are a special type of associative memories based on the **median** operator.

Two kind of associative memories were proposed in:

**H. Sossa et al. (2004). New Associative Memories to Recall Real-Valued Patterns. LNCS 3287. Springer Verlag. Pp. 195-202.**

**H. Sossa et al. (2005). Median Associative Memories: New Results. LNCS 3773. Springer Verlag. Pp. 1036-1046.**

One hetero-associative and one auto-associative.

Only hetero-associative case is studied.

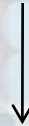
# BASICS ON MEDIAN ASSOCIATIVE MEMS:

Functioning of the median operator: To select the mean value we do the following:

Suppose we have the following series of non-ordered numbers:

5 1 2 4 7 0 3 2 3

1. We **proceed** to order them:



0 1 2 2 3 3 4 5 7

Instead of a 7, we have now a 3 as the median value.

2. We **select** as new value the mean value of this new ordered series:



0 1 2 2 3 3 4 5 7

3. We **take** this number as the value.

# BASICS ON MEDIAN ASSOCIATIVE MEMS:

One hetero-associative memory is described. Let us call HAM-memory of type **M**. TRAINING PHASE:

**Step 1:** For each  $\xi=1,2,\dots,p$ , from each couple  $(\mathbf{y}^\xi, \mathbf{x}^\xi)$

build matrix:

$$\mathbf{M}^\xi = \mathbf{y} \diamond_A \mathbf{x}^t = \begin{pmatrix} A(y_1, x_1) & A(y_1, x_2) & \cdots & A(y_1, x_n) \\ A(y_2, x_1) & A(y_2, x_2) & \cdots & A(y_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ A(y_m, x_1) & A(y_m, x_2) & \cdots & A(y_m, x_n) \end{pmatrix}_{m \times n}$$

$$A(y_i^\xi, x_j^\xi) = y_i^\xi - x_j^\xi$$

**Step 2:** Obtain matrix **M** as:  $\mathbf{M} = \mathbf{med}_{\xi=1}^p [\mathbf{M}^\xi]$

The  $ij$ -th component **M** is given as  $m_{ij} = \mathbf{med}_{\xi=1}^p A(y_i^\xi, x_j^\xi)$

# BASICS ON MEDIAN ASSOCIATIVE MEMS:

RECALLING PHASE: We have two cases, i.e.:

**Case 1:** Recall of a fundamental pattern  $\mathbf{y}^w$ . A pattern  $\mathbf{x}^w$ , with  $w \in \{1, 2, \dots, p\}$  is presented to the memory  $\mathbf{M}$  and the following operation is done:

$$\mathbf{M} \diamond_{\mathbf{B}} \mathbf{x}^w$$

The result is a column vector of dimension  $n$ , with  $i$ -th component given as:

$$\left( \mathbf{M} \diamond_{\mathbf{B}} \mathbf{x}^w \right)_i = \mathbf{med}_{j=1}^n \mathbf{B}(m_{ij}, x_j^w) \quad \mathbf{B}(y_i^\xi, x_j^\xi) = y_i^\xi + x_j^\xi$$

# BASICS ON MEDIAN ASSOCIATIVE MEMS:

**Case 2:** Recall of a pattern from an altered version of its key.

A pattern  $\tilde{\mathbf{x}}$  (altered version of a pattern  $\mathbf{x}^w$ ) is presented to the auto-associative memory  $\mathbf{M}$  and the following operation is done:

$$\mathbf{M} \diamond_{\mathbf{B}} \tilde{\mathbf{x}}$$

Again, the result is a column vector of dimension  $n$ , with  $i$ -th component given as:

$$\left(\mathbf{M} \diamond_{\mathbf{B}} \tilde{\mathbf{x}}\right)_i = \mathbf{med}_{j=1}^n \mathbf{B}(m_{ij}, \tilde{x}_j)$$

$$\mathbf{B}(y_i^\xi, x_j^\xi) = y_i^\xi + x_j^\xi$$



# BASICS ON MEDIAN ASSOCIATIVE MEMS:

A list of formal propositions under which a MED-MEMS works, can be found in:

**H. Sossa et al. (2004). New Associative Memories to Recall Real-Valued Patterns. LNCS 3287. Springer Verlag. Pp. 195-202.**

**H. Sossa et al. (2005). Median Associative Memories: New Results. LNCS 3773. Springer Verlag. Pp. 1036-1046.**

For their correct functioning, in the general case, MED-MEMS need to use the **transformation method** reported in:

**H. Sossa et al. (2004). Transforming Fundamental Set of Patterns to a Canonical Form to Improve Pattern Recall. LNAI 3315. Pp. 687-696.**

# BASICS ON MEDIAN ASSOCIATIVE MEMS:

Median MEMs have been applied in:

1. Pattern restoration.
2. Object classification.
3. Object classification under occlusions.

# BEHAVIOR OF THE MED-AMS:

In this paper a behavioral study of the MED-HAM using true-color noisy patterns is presented.

The benchmark used in this set of experiments is composed by 14,440 color images of 63 X 63 pixels and 24 bits in a bmp format:

H. Sossa and R. A. Vazquez: Flower and Animals Database, Available in <http://roberto.a.vazquez.googlepages.com>

This benchmark is composed of 40 classes of flowers and animals.




# BEHAVIOR OF THE MED-AMS:

Per each class:

- 90 images were altered with **additive noise** (from 0% of the pixels to 90% of the pixels),
- 90 images were altered with **subtractive noise** (0% of the pixels to 90% of the pixels),
- 90 images were altered with **mixed noise** (0% of the pixels to 90% of the pixels) and
- 90 images were altered with **Gaussian noise** (0% of the pixels to 90% of the pixels).
- In addition, one image of each class was altered **by removing** some parts of the image.

# BEHAVIOR OF THE MED-AMS:

Additive noise					
Subtractive noise					
Mixed noise					
Gaussian noise					
	10 % of noise	20 % of noise	30 % of noise	40 % of noise	50 % of noise
Missing data					

Some images from the benchmark used to train and test the MED-AM. **Details of how the images were generated can be found in the paper.**

# BEHAVIOR OF THE MED-AMS:

## Training:

Once the images were transformed into vectors, a MED-HAM was trained using a set of associations composed by the 40 image patterns which are not altered with any kind of noise.

# BEHAVIOR OF THE MED-AMS:

## Testing:

First to all, we verified if the MED-HAM was able to recall the complete set of associations. **The whole FS was correctly recalled.**

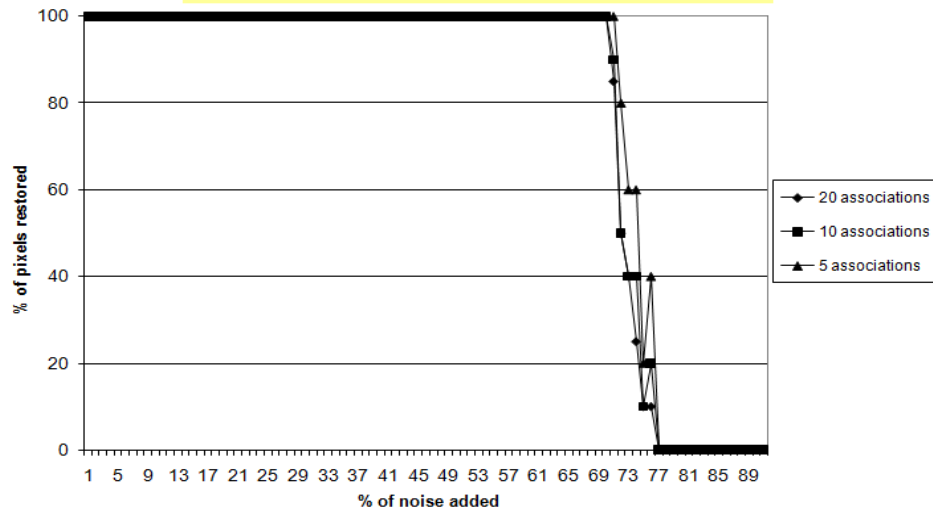
Then we verified the behavior of MED-HAM using noisy versions of the images used to train it.

After that, we performed a study concerning on how the number of associations influences the behavior of the MED-HAM.

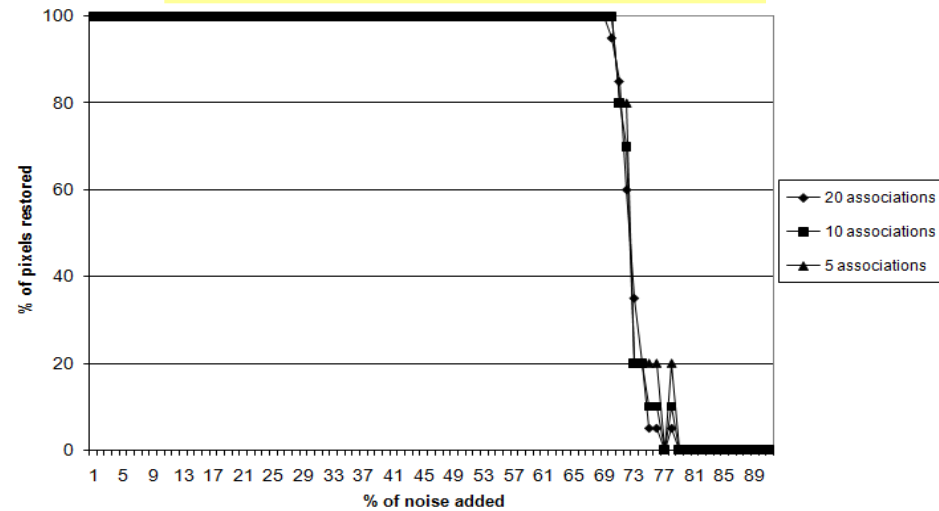
In order to measure the accuracy of the MED-HAM we counted the percentage of pixels correctly recalled.

# BEHAVIOR OF THE MED-AMS:

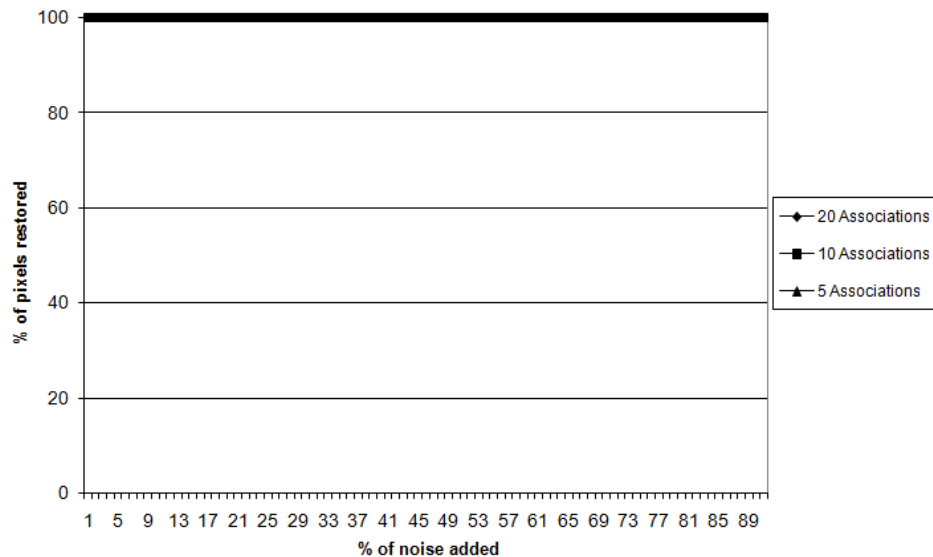
## Additive noise



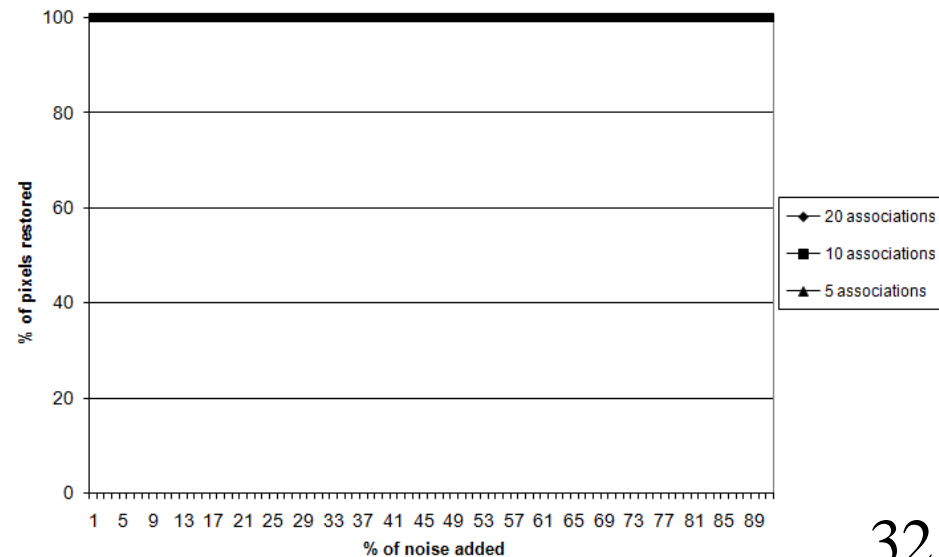
## Subtractive noise



## Mixed noise



## Gaussian noise



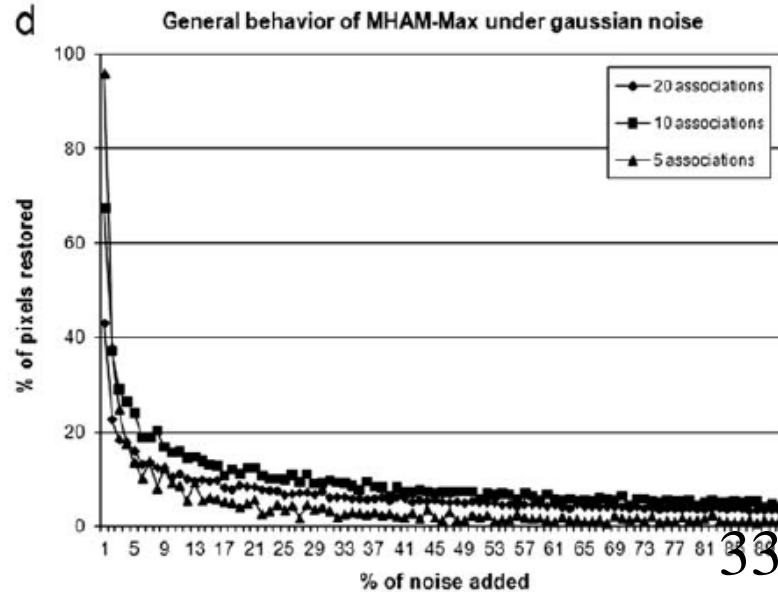
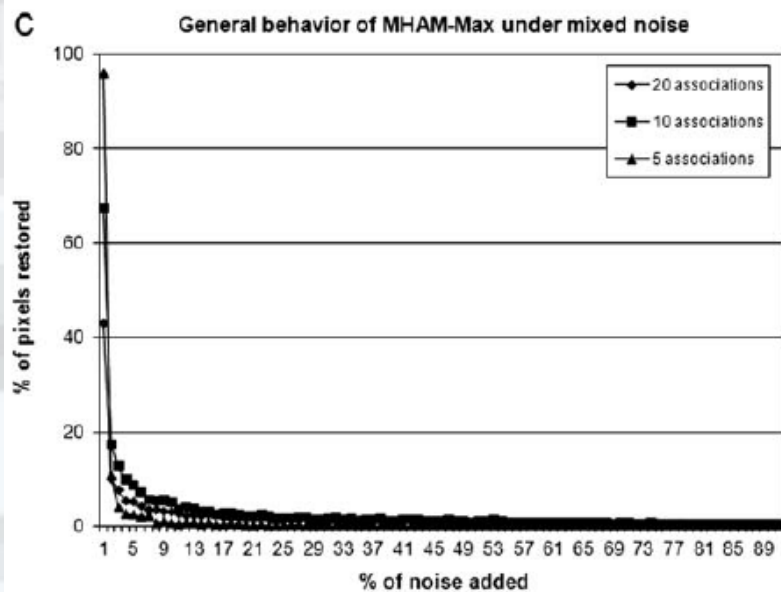
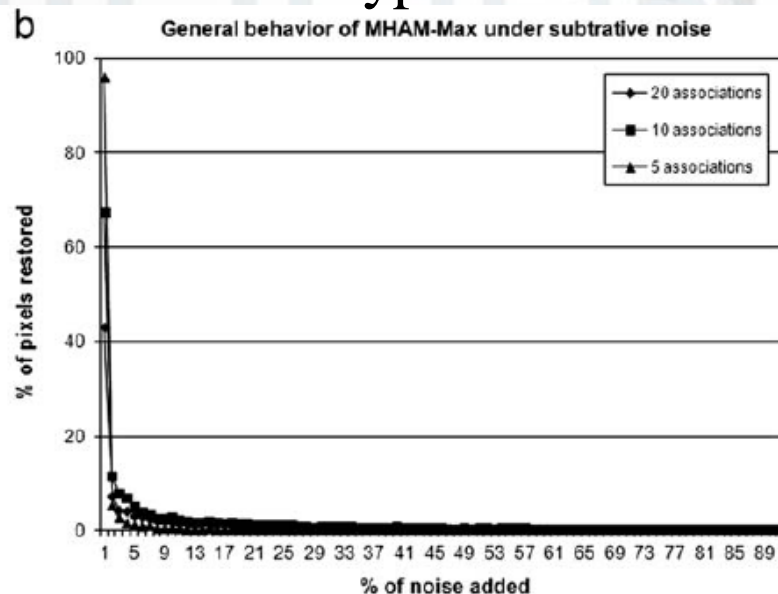
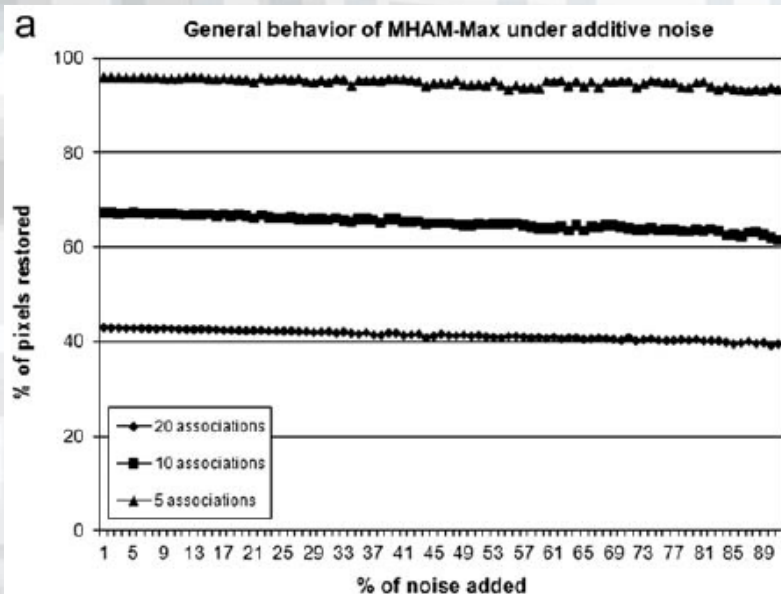


# BEHAVIOR OF MORPHOLOGICAL ASS MEMS:

General behavior of the MHAM-Max under different type of noises.

The MHAM-Max was not able to recall the complete set of associations.

Even when patterns were not altered with noise, the MHAM-Max correctly recalled only 43.1% of the pixels.



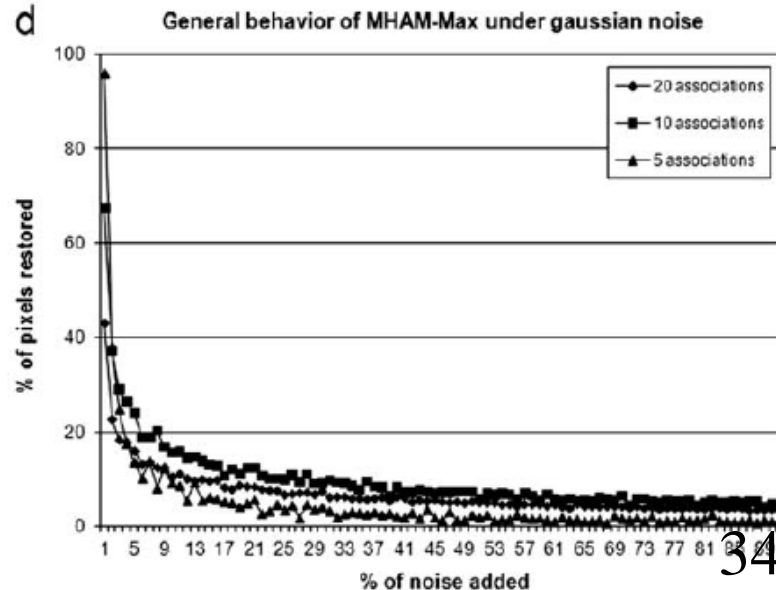
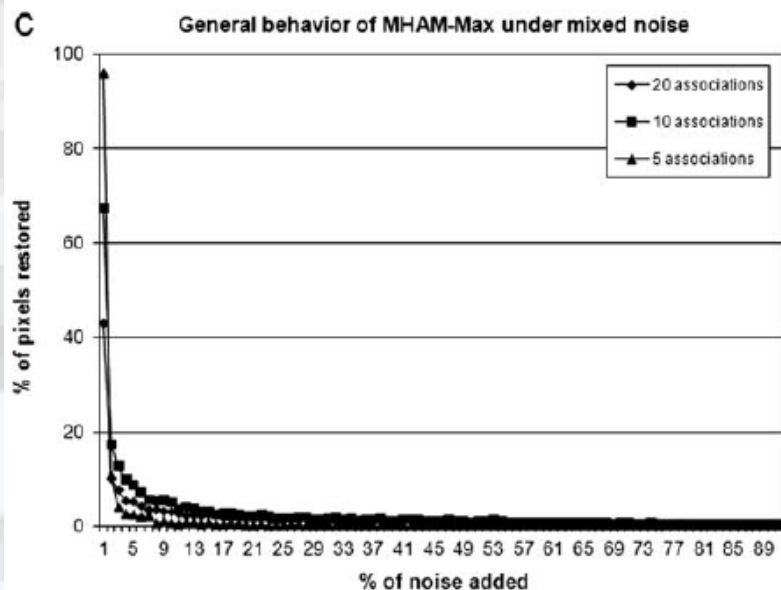
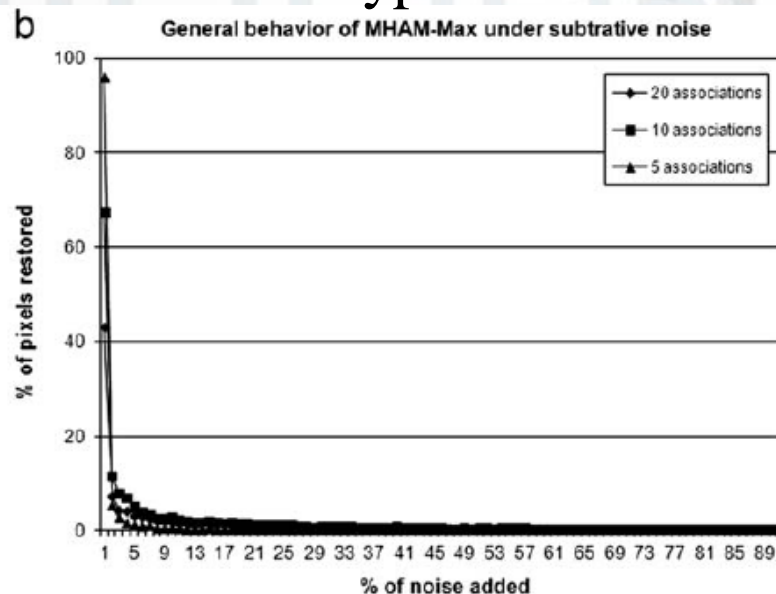
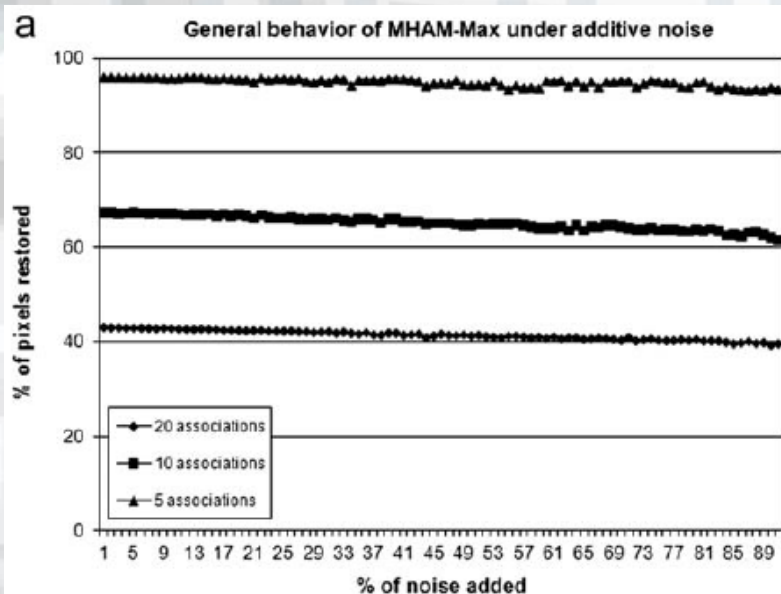
# BEHAVIOR OF MORPHOLOGICAL ASS MEMS:

General behavior of the MHAM-Max under different type of noises.

The accuracy of the MHAM-Max increases as the number of associations decreases.

This fact holds only for patterns altered with **additive** noise.

For the other type of noises, the accuracy decreases.

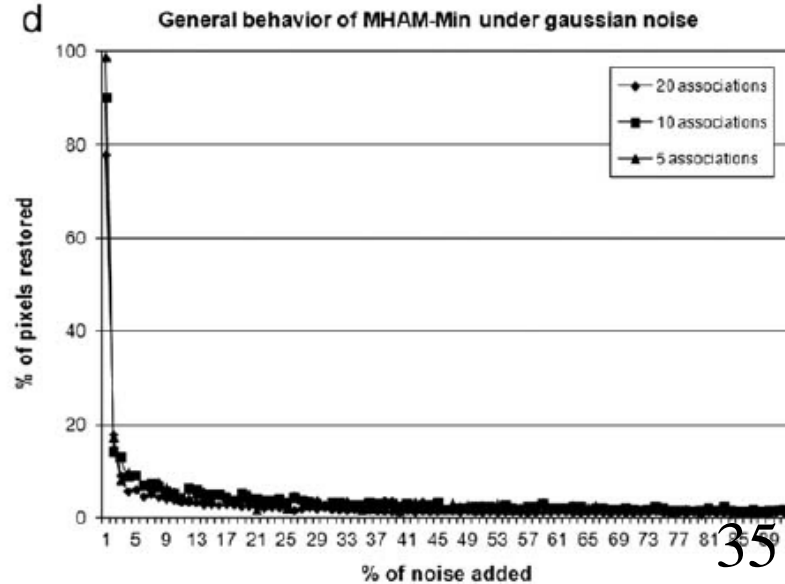
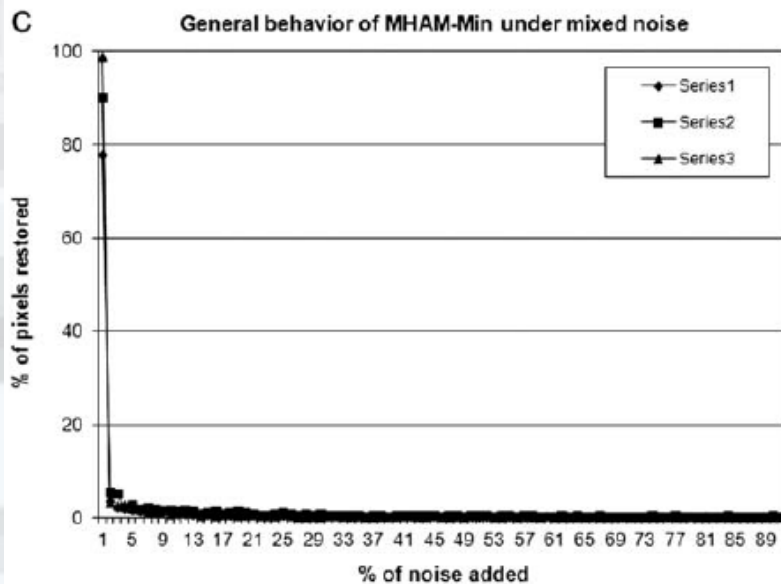
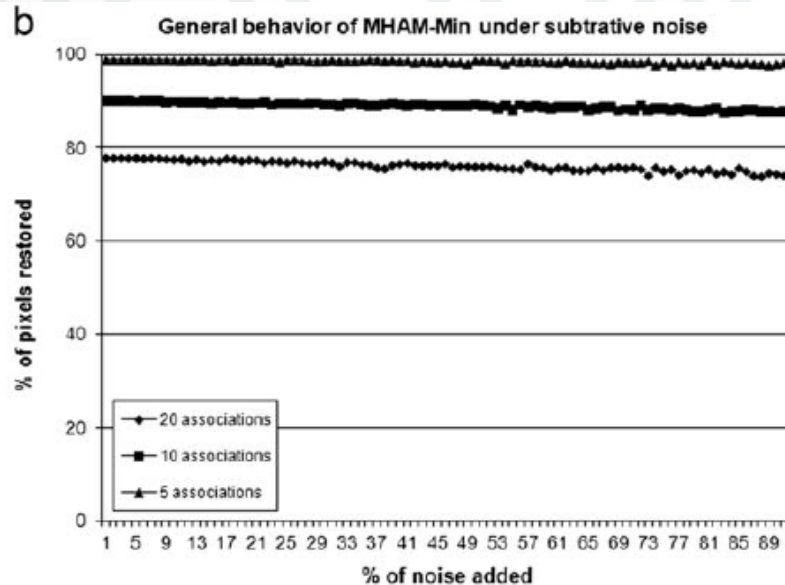
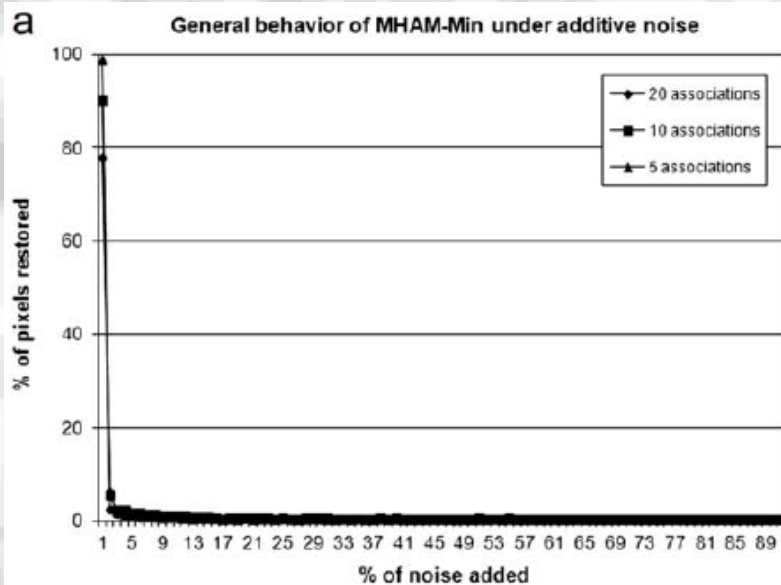


# BEHAVIOR OF MORPHOLOGICAL ASS MEMS:

General behavior of the MHAM-Min under different type of noises.

The MHAM-Min was not able to recall the complete set of associations.

Even when patterns were not altered with noise, the MHAM-Min correctly recalled only 77.6% of the pixels.



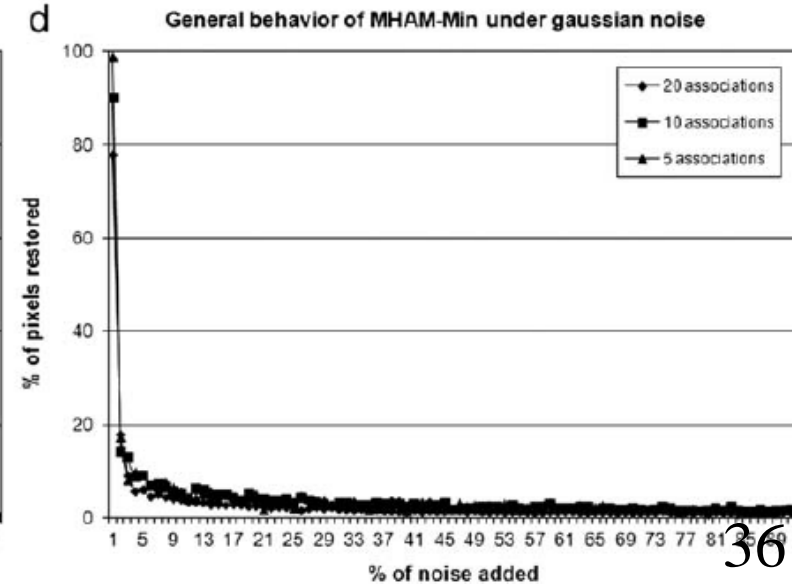
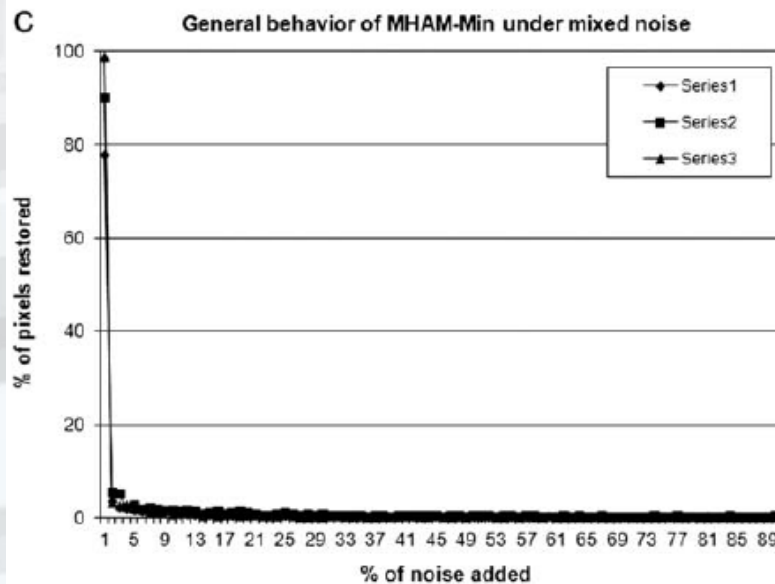
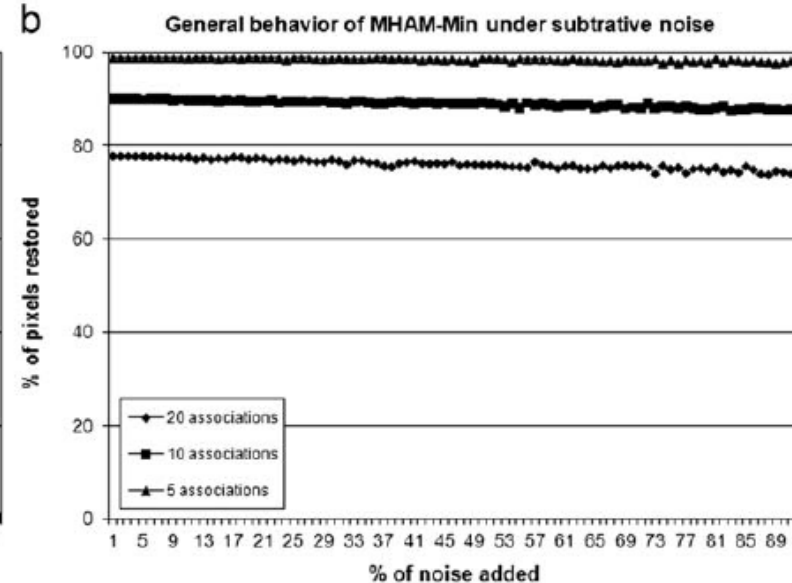
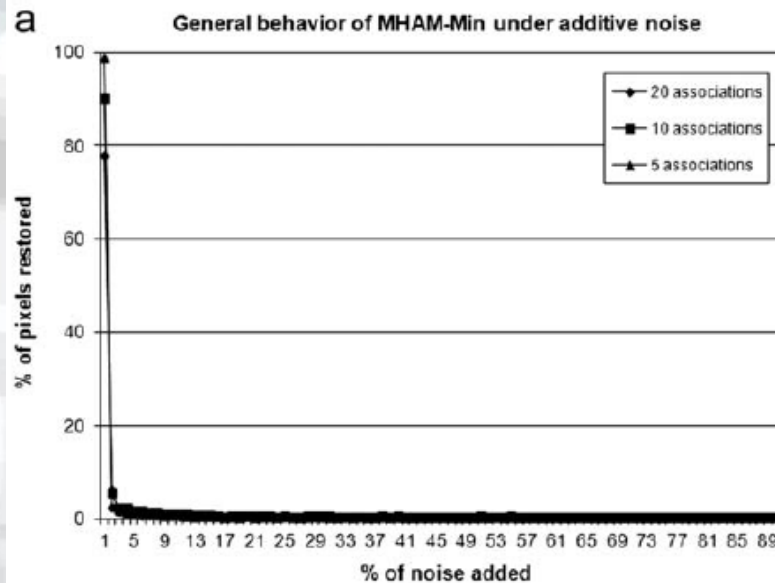
# BEHAVIOR OF MORPHOLOGICAL ASS MEMS:

General behavior of the MHAM-Min under different type of noises.

The accuracy of the MHAM-Min increases as the number of associations decreases.

This fact holds only for patterns altered with **subtractive** noise.

For the other type of noises, the accuracy decreases.



# BEHAVIOR OF THE MED-AMS:

Kind of Noise	MAX-MHAM	MIN-MHAM	MED-AMS
Additive	68.00%	2.00%	73.00%
Subtractive	3.00%	88.00%	73.00%
Mixed	1.50%	1.50%	100.00%
Gaussian	5.50%	3.00%	100.00%

# A REAL APPLICATION: IMAGE CATEGORIZATION

Image categorization is not a trivial problem when pictures are taken from real life situations.

An initial effort was reported in:

R. A. Vazquez and H. Sossa (2006). Associative memories applied to image categorization. LNCS 4225, pp. 549-558.

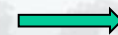
# A REAL APPLICATION: IMAGE CATEGORIZATION

The idea is as follows: If a MED-HAM is fed with a picture, we expect that the MEMORY responds (for example) with a word indicating the content of the picture.

For example, if the picture contains a tiger, we would expect that the Memory should respond with the word “**tiger**”.



MED-AM



TIGER

# A REAL APPLICATION: IMAGE CATEGORIZATION



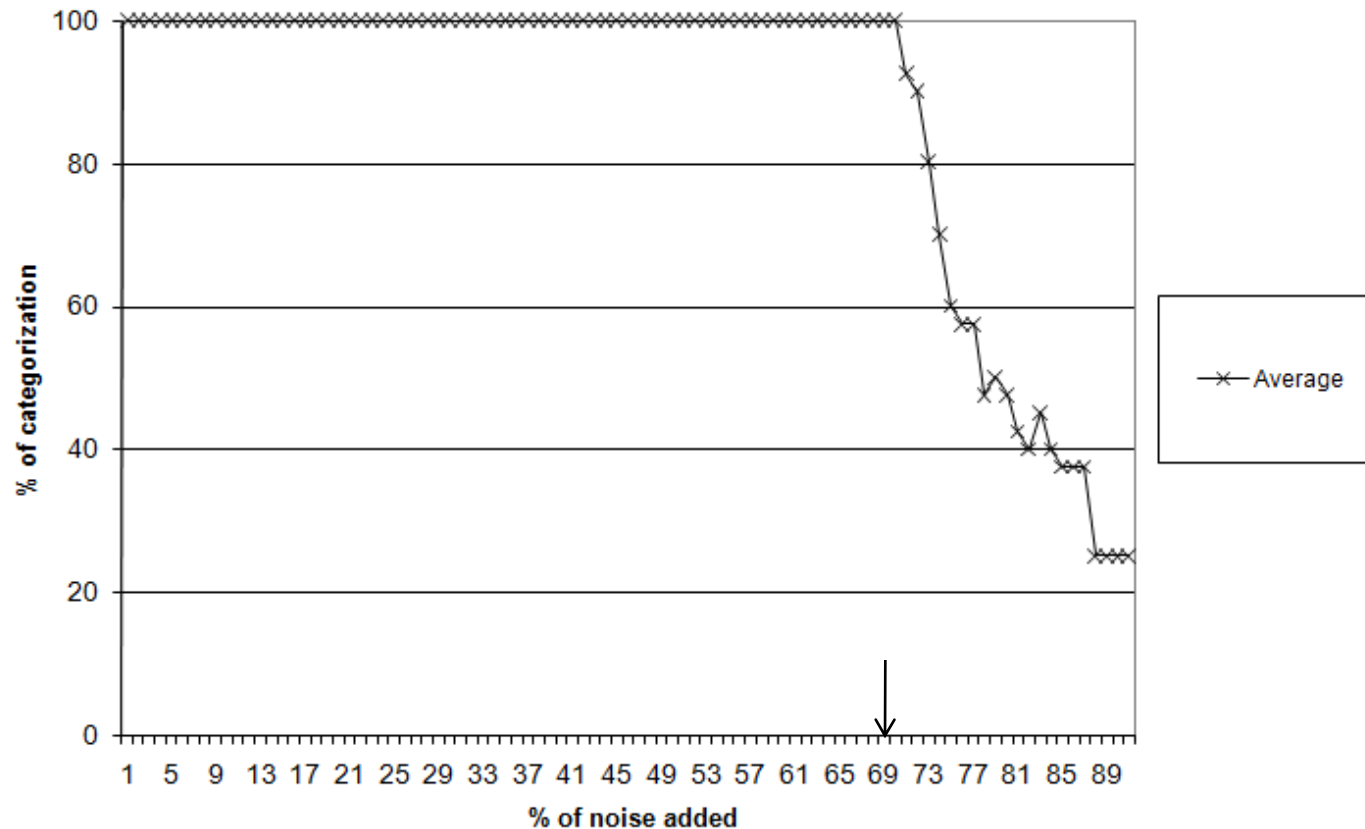
Fundamental set of associations composed of 40 associations used to train the MED-HAM applied to the image categorization problem.

In average, the accuracy of the memories for the image categorization task was of **88.27%**.



# A REAL APPLICATION: IMAGE CATEGORIZATION

Image categorization of MED-HAMs in the presence of additive noise



If the level of noise added to an image is less than 70%, all the images are correctly categorized or classified. If the quantity of noise surpasses this threshold, the accuracy starts to decrease.

# CONCLUSIONS AND PRESENT RESEARCH:

## CONCLUSIONS:

MED-HAMs are not sensitive to the amount of noise. After 73% of additive and subtractive noise, the accuracy of the model tends to decrease.

We also observed that the model is robust to mixed noise and Gaussian noise too.

Concerning image categorization problem, the accuracy of the MEDMEMS in average was of 88.27%.

The efficiency of the memory tends to decrease when the added noise is more than 70%.

## PRESENT RESEARCH:

Automatic generation of associative memories by means of genetic programming.



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# ¡ THANKS !



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# BEHAVIOR OF MAMs:

