A genetic algorithm for solving the generalized vehicle routing problem

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Outline

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic.
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
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Generalized combinatorial optimization problems

- Classical combinatorial optimization problems can be generalized by considering a related problem relative to a given partition of the nodes of a graph into sets of nodes (clusters).
- In this way, it is introduced the class of generalized combinatorial optimization problems:
  - generalized traveling salesman problem,
  - generalized vehicle routing problem,
  - generalized minimum spanning tree problem,
  - generalized minimum edge-biconnected network problem,
  - generalized fixed-charge network design problem, etc.
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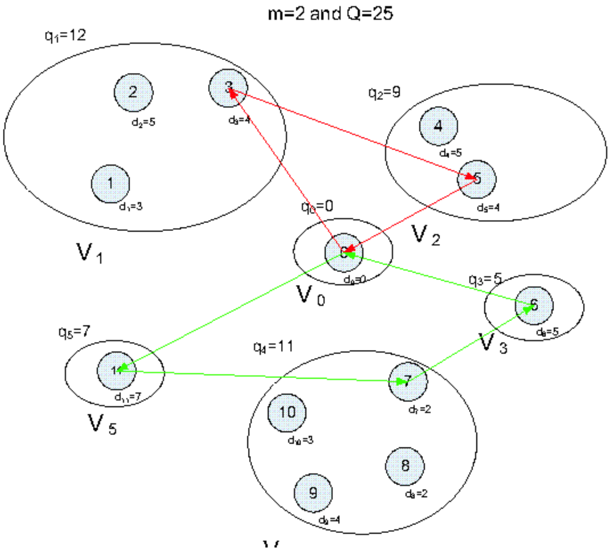
The Generalized Vehicle Routing Problem (GVRP)

- The GVRP was introduced by Ghiani and Improta [1].
- The goal of the problem is to design the optimal delivery or collection routes, subject to capacity restrictions, from a given depot to a number of predefined, mutually exclusive and exhaustive node-sets (clusters).
- Kara and Bektas [2] proposed an integer programming formulation for GVRP with a polynomially increasing number of binary variables and constraints.
- The GVRP reduces to the classical VRP when all the clusters are singletons and to the GTSP when $m = 1$ and $Q = \infty$. The GVRP is *NP*-hard because it includes the GTSP as a special case when $m = 1$ and $Q = \infty$.
- An illustrative scheme of the GVRP and a feasible tour is shown in the next figure.

The Generalized Vehicle Routing Problem (GVRP)



Definition of the Generalized Vehicle Routing Problem

- Let $G = (V, A)$ be a directed graph with $V = \{0, 1, 2, ..., n\}$ as the set of vertices and the set of arcs $A = \{(i, j) \mid i, j \in V, i \neq j\}$. A nonnegative cost c_{ij} associated with each arc $(i, j) \in A$. The set of vertices is partitioned into $k + 1$ mutually exclusive nonempty subsets, called clusters, $V_0, V_1, ..., V_k$.
- Each customer has a certain amount of demand and the total demand of each cluster can be satisfied via any of its nodes. There exist m identical vehicles, each with a capacity Q.
- The GVRP consists in finding the minimum total cost tours of starting and ending at the depot, such that each cluster should be visited exactly once, the entering and leaving nodes of each cluster is the same and the sum of all the demands of any tour (route) does not exceed the capacity of the vehicle Q.

Solving the Generalized Vehicle Routing Problem

- an efficient transformation of the GVRP into a Capacitated Arc Routing Problem (CARP) (Ghiani and Improta [1]);
- an ant colony based algorithm (Pop et al. [4]), sensitive ant models (Pintea et al. [3]);
- an efficient transformation of the generalized vehicle routing problem into the vehicle routing problem (Pop [5]);
- heuristic and metaheuristic algorithms:
  - constructive heuristics: Nearest Neighbour and a Clarke-Wright based heuristic;
  - improvement heuristics: String Cross (SC), String Exchange (SE), String Relocation (SR) and String Mix (SM);
  - a local-global heuristic;
  - a genetic algorithm.

Heuristic algorithms for solving the GVRP

Nearest Neighbor

In this algorithm the rule is always to go next to the nearest unvisited customer subject to the following restrictions:

- we start from the depot,
- from each cluster is visited exactly one vertex (customer) and
- the sum of all the demands of the current tour (route) does not exceed the capacity of the vehicle Q.

If the sum of all the demands of a current tour (route) exceeds the capacity of the vehicle then we start again from the depot and visit next the nearest customer from an unvisited yet cluster.
If all the clusters are visited, then the algorithm terminates.

Heuristic algorithms for solving the GVRP

A Clarke-Wright based heuristic algorithm

- **Step 1 (Savings computation).** For each $i \in V_l$ and $j \in V_p$, where $l \neq p$ and $l, p \in \{1, ..., k\}$ compute the savings:

$$s_{ij} = c_{i0} + c_{0j} - c_{ij}$$

At the beginning we create k routes denoted $(0, i_l, 0)$, $l \in \{1, ..., k\}$ as follows for each cluster V_l we define

$$c_{0i_l} = \min\{c_{0j} \mid j \in V_l\}.$$

There will be as many routes as the number of clusters and total distance of the routes is:

$$d = c_{0i_1} + c_{0i_2} + ... + c_{0i_k}.$$

Heuristic algorithms for solving the GVRP

A Clarke-Wright based heuristic algorithm

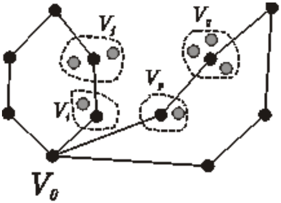
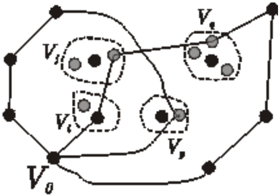
- **Step 2 (Route extension).**
  - Consider in turn each route $(0, i, ..., j, 0)$.
  - Determine the first saving s_{ui} or s_{jv} that can feasibly be used to merge the current route with another route ending with $(u, 0)$ or starting with $(0, v)$, for any $u \in V_l$ and $v \in V_p$, where $l \neq p$ and $l, p \in \{1, ..., k\}$ and V_l and V_p are clusters not visited by the route $(0, i, ..., j, 0)$.
  - Because at a given moment there can exist more feasible route extensions, the priority will have that one that produces the biggest reduction of the total distance of the route.
  - We implement the merge and repeat this operation to the current route. If no feasible merge exists, consider the next route and reapply the same operations.
- Stop when no route merge is feasible.

Heuristic algorithms for solving the GVRP

Improvement heuristics

The improvement heuristics algorithms for the GVRP are based on simple routes modifications and may operate on each vehicle route taken separately, or on a several routes at a time.

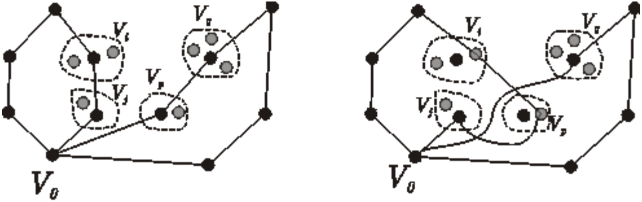
- *String cross (SC)*: two strings of vertices are exchanged by crossing two edges of two different routes.

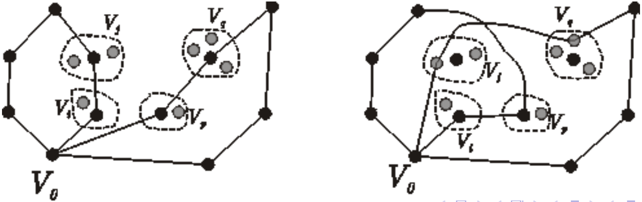
Heuristic algorithms for solving the GVRP

Improvement heuristics

- *String exchange (SE)*: two strings of at most *r* vertices are exchanged between two routes.



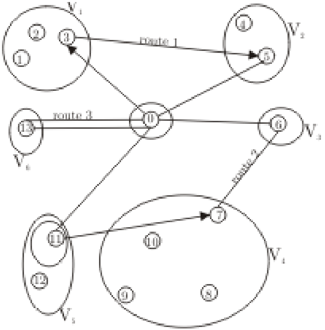
- *String relocation (SR)*: a string of at most *k* vertices is moved from one route to another ($k = 1$ or $k = 2$).



A Genetic Algorithm for Solving the GVRP

The idea behind GA is to model the natural evolution by using genetic inheritance together with Darwins theory.

In GA, the *population* consists of a set of *solutions* or *individuals* instead of *chromosomes*. A *crossover operator* plays the role of reproduction and a *mutation operator* is assigned to make random changes in the solutions.



A Genetic Algorithm for Solving the GVRP

Representation

- We represent a chromosome by an array so that the gene values correspond to the nodes selected to form the collection of generalized routes.
- In previous figure, we plot an individual representing a possible solution for a GVRP instance with 13 customers partitioned into 5 clusters using 3 vehicles at most. The values $\{1, ..., 13\}$ represent the customers while $\{0\}$ is the route splitter. Route 1 begins at the depot then visits customers 3 and 5 belonging to the clusters V_1, respectively V_2 and returns to the depot. Route 2 starts at the depot and visits the customers 6-7-11 belonging to the clusters $V_3 - V_4 - V_5$. Finally, in route 3 only customer 13 from the cluster V_6 is visited.

A Genetic Algorithm for Solving the GVRP

Initial population

- The construction of the initial population is of great importance to the performance of GA, since it contains most of the material the final best solution is made of.
- In our algorithm, we have produced 20 initial solutions generated randomly: by selecting randomly the nodes from each clusters and the collection of generalized routes, but they can also be results of some construction methods.

The fitness value

- Every solution has a fitness value assigned to it, which measures its quality. In our case the, the fitness value of a GVRP is given by the total cost of travelling for all the vehicles, i.e. the objective function of the following integer programming model.

An integer programming formulation

$$\text{minimize} \quad \sum_{v\in M}\sum_{(i,j)\in A} c_{ij}x_{ij}^v$$

$$\text{subject to} \quad \sum_{i\in V_l} z_i = 1, \quad \text{for } l = 1, ..., k$$

$$\sum_{v\in M}\sum_{j\in V} x_{ij}^v = z_i, \ \forall i \in \{1, ..., n\}$$

$$\sum_{i\in V\setminus\{0\}} d_i \sum_{j\in V} x_{ij}^v \le Q, \ \forall v \in M$$

$$\sum_{i\in V\setminus\{0\}} x_{0j}^v = 1, \ \forall v \in M$$

$$\sum_{i\in V} x_{ik}^v - \sum_{j\in V} x_{kj}^v = 0, \ \forall k \in V \setminus \{0\} \text{ and } \forall v \in M$$

$$x_{ij}^v, z_i \in \{0,1\}, \ \forall i \in V \ \forall (i,j) \in A, v \in M$$

A Genetic Algorithm for Solving the GVRP

Crossover operator

- Two parents are selected from the population by the binary tournament method, i.e. the individuals are chosen from the population at random.
- Offspring are produced from two parent solutions using the following 2-point order crossover procedure: it creates offspring which preserve the order and position of symbols in a subsequence of one parent while preserving the relative order of the remaining symbols from the other parent.
- It is implemented by selecting two random cut points which define the boundaries for a series of copying operations. First, the symbols between the cut points are copied from the first parent into the offspring. Then, starting just after the second cut-point, the symbols are copied from the second parent into the offspring, omitting any symbols that were copied from the first parent.

A Genetic Algorithm for Solving the GVRP

Example Crossover operator

We assume two well-structured parents chosen randomly, with the cutting points between nodes 2 and 3, respectively 5 and 6:

$P_1 = 13\ \ 0\ |\ 3\ \ 5\ \ 0\ |\ 11\ \ 7\ \ 6$

$P_2 = 4\ \ 2\ |\ 13\ \ 0\ \ 11\ |\ 10\ \ 6$

the cluster representation of the parents is as follows:

$C_1 = 6\ \ 0\ |\ 1\ \ 2\ \ 0\ |\ 5\ \ 4\ \ 3$

$C_2 = 2\ \ 1\ |\ 6\ \ 0\ \ 5\ |\ 4\ \ 3$

The sequences between the two cutting-points are copied into the two offspring:

$O_1 = x\ \ x\ |\ 3\ \ 5\ \ 0\ |\ x\ \ x\ \ x$

$O_2 = x\ \ x\ |\ 13\ \ 0\ \ 11\ |\ x\ \ x$

The offspring generated using the proposed crossover operator are:

$O_1 = 13\ \ 0\ |\ 3\ \ 5\ \ 0\ |\ 11\ \ 10\ \ 6$

$O_2 = 0\ \ 3\ |\ 13\ \ 0\ \ 11\ |\ 5\ \ 0\ \ 7\ \ 6$

A Genetic Algorithm for Solving the GVRP

Mutation operator

We use in our GA two random mutation operators:

- the first one (intra-route mutation) selects randomly a cluster to be modified and replaces its current node by another one randomly selected from the same cluster
- the second one (inter-route mutation) is a swap operator, it picks two random locations in the solution vector and swaps their values.

The new chromosome is accepted if it results in a feasible GVRP.

The developed GA uses the steady-state approach, in which eligible offspring enter the population as soon as they are produced, with inferior individuals being removed at the same time, so that the size of the population remains constant.

A Genetic Algorithm for Solving the GVRP

Computational results

- The performance of the proposed GA for *GVRP* was tested on seven benchmark problems drawn from *TSPLIB*.
- The testing machine was an Intel Dual-Core 1,6 GHz and 1 GB RAM. The operating system was Windows XP Professional. The algorithm was developed in Java.
- In the genetic algorithm for *GVRP*, the values of the parameters were chosen as follows: population size 20, the number of offspring 40, the maximum number of generations 100, the intra-route mutation rate 5% and the inter-route mutation rate 5%.

A Genetic Algorithm for Solving the GVRP

Computational results

In the next tables are shown the computational results obtained for solving the *GVRP* using the proposed **GA** algorithm comparing with the **ACS** algorithm [5].

Table 1. Best Values and Times - *ACS* and *GA* algorithms for *GVRP*

Problem	*ACS*	*Time ACS*	*GA*	*Time GA*
11eil51	418.85	212	237.00	7
16eil76A	668.78	18	583.80	18
16eil76B	625.83	64	540.87	95
16eil76C	553.21	215.00	336.45	50
16eil76D	508.81	177.00	295.55	12
21eil101A	634.74	72	476.98	38
21eil101B	875.58	8.00	664.45	55

Conclusions and future work

- The aim of this paper was to present heuristic and metaheuristic algorithms for solving the GVRP:
  - constructive heuristics;
  - improvement heuristics;
  - a genetic algorithm.
- Hybridizing exact methods and metaheuristics
  - use exact algorithms to explore large neighborhoods in local search algorithms
  - use information from relaxations of integer programming problems to guide local search or constructive algorithms
  - use exact algorithms for specific procedures in hybrid metaheuristics.

For Further Reading I

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- I. Kara and T. Bektas, Integer linear programming formulation of the generalized vehicle routing problem, in Proc. of the 5-th EURO/INFORMS Joint International Meeting, 2003.
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- P.C. Pop, C.M. Pintea, I. Zelina and D. Dumitrescu, Solving the Generalized Vehicle Routing Problem with an ACS-based Algorithm, American Institute of Physics (AIP), Conference Proceedings: BICS 2008, Vol.1117, No.1, 157-162, 2009.

For Further Reading II

P.C. Pop, Efficient Transformations of the Generalized Combinatorial Optimization Problems into Classical Variants, 9-th Balkan Conference on Operational Research BALCOR 2009, 2-6 September, Constanta, Romania.