# Fusion of fuzzy spatial relations

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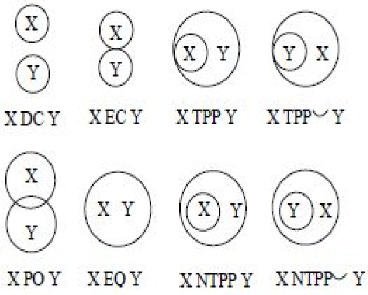
Hybrid Artificial Intelligence Systems.
San Sebastián, SPAIN
June 22-25, 2010

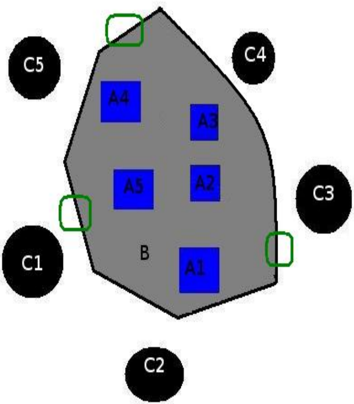
# Outline

- Image understanding
- Automatic image interpretation
- Spatial reasoning
- Spatial prediction
- Semantic web

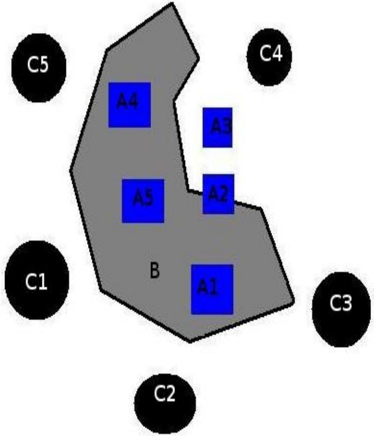
- Topological relations provides us geometrical structure of image.
- Relations like Disjoin D, Externally Connected EC, Partially Overlap PO, etc., are defined in *RCC* [a] theory or equivalently with point set topology [b].
- Both the theories are extended to deal fuzziness at object level.




*a*. Randell et al.-1992
*b*. Max J. Egenhofer and R D Franzosa-1991




- Every green box, blue box and black circle has the same topological relation with gray object *B*.
- Where a topological relation exist ?
- Object geometry becomes important for directional relations




- Qualitative directional relations don't distinguish between topological relations.
- Internal Cardinal Directions relations (ICD) method is applied to know the object position inside the extended reference object.
- Fuzzy directional relations works only for disjoint objects.

- Fuzziness at object level or relations level ?
- Where a topological relation exist ?
- Object geometry ?
- Fuzzy directional relations method work mostly for disjoint objects.

**Does there exist a single method which can answer all these questions ?**

$A = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$, { before, meet, overlap, start, finish, during, equal } and their inverses.

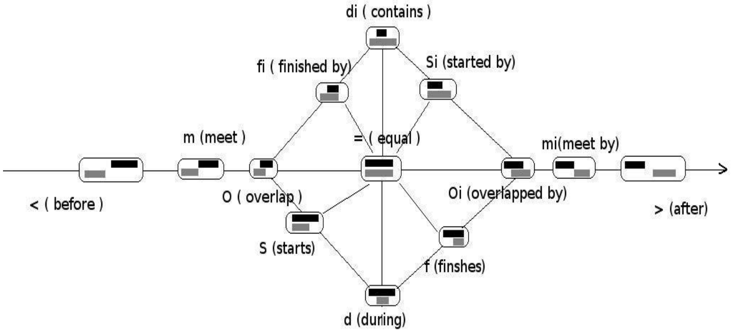



**FIGURE:** Black segment represents the reference object and grey segment represents argument object

1. J.F. Allen -1983

- Apply temporal Allen relations in $1D$ spatial domain [2].
- Force histograms [3] $\boxed{\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv}$
- $F : \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+$.
  $\boxed{F(\theta, A_\theta(v), B_\theta(v)) = \sum_{k=1..n, j=1...m} f(x_{Ii}, y_{IiJj}^{\theta}, z_{Jj})}$

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2. J. Malki et al. 2002
3. P. Matsakis and L. Wendling-1999

**f- histogram** $f : (\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+) \rightarrow \mathbb{R}_+$ and

$$\boxed{f(x_I, y_{IJ}^{\theta}, z_J) = \int_{x_I+y_{IJ}^{\theta}}^{x_I+y_{IJ}^{\theta}+z_J} \int_0^{z_J} \phi(u-w)dwdu}$$

The Force histograms depends upon definition of the function $\phi$.
$\mathbf{F}^{AB}(\theta)$ is a real valued function.

**$\phi$- histogram** $\phi : \mathbb{R} \rightarrow \mathbb{R}_+$ such that, $r \in \mathbb{R}$.

$$\phi_r(y) = \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

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$\mathbf{F}^{AB}(\theta)$ is a real valued function.

- Fuzzy membership function assign a membership value to a relation. $\mu : D \rightarrow [0,1]$ where $D$ is the distance between two neighboring relations.

$$\mu(x;\alpha,\beta,\gamma,\delta) = max(min(\frac{x-\alpha}{\beta-\gamma}, 1, \frac{\delta-x}{\delta-\gamma}), 0)$$

## Fuzzy Allen relations [4]

- $f_<(I,J) = \mu_{(-\infty,-\infty,-b-3a/2,-b-a)}(y)$
- $f_m(I,J) = \mu_{(-b-3a/2,-b-a,-b-a,-b-a/2)}(y)$
- $f_{mi}(I,J) = \mu_{(-a/2,0,0,a/2)}(y)$
- $f_o(I,J) = \mu_{(-b-a,-b-a/2,-b-a/2,b)}(y)$
- $f_f(I,J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))$
- $f_{di}(I,J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))$

where $a = min(x,z)$, $b = max(x,z)$, $x$ and $z$ is the length of longitudinal section of object $A$ and $B$.

4. Matsakis and Nikitenko 2005

# Couple force histogram to fuzzy Allen relations

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, A_\theta(v), B_\theta(v))dv$$

and

$$F_r(\theta, A_\theta(v), B_\theta(v)) = r(I, J)$$

# Treatment of longitudinal section

Recall definition of force histogram for treatment of longitudinal section.

$$F(\theta, A_\theta(v), B_\theta(v)) = \sum_{k=1..n, j=1...m} f(x_{Ii}, y^{\theta}_{IiJj}, z_{Jj})$$

Replace $\sum$ by a fuzzy operator.

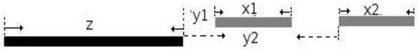
$$F(\theta, A_\theta(v), B_\theta(v)) = \odot(f(x_1, y^{\theta}_1, z), f(x_2, y^{\theta}_2, z), ...., f(x_n, y^{\theta}_n, z))$$

Where $\odot$ is a fuzzy operator.

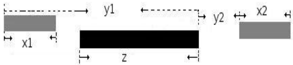
An operator :
$T : [0,1] \times [0,1] \rightarrow [0,1]$

- $\mu_{(OR)}(u) = max(\mu_{(A)}(u), \mu_{(B)}(u))$.
- $\mu_{(AND)}(u) = min(\mu_{(A)}(u), \mu_{(B)}(u))$.
- $\mu_{(PROD)}(u) = \Pi_{i=1}^{2}(\mu_{(i)}(u))$.
- $\mu_{(SUM)}(u) = 1 - \Pi_{i=1}^{2}(\mu_{(i)}(u))$.
- $\mu_{(\gamma)}(u) = [\mu_{(SUM)}(u)]^{\gamma} * [(\mu_{(PROD)}(u)]^{1-\gamma}$ where $\gamma \in [0,1]$.
- .
- .

$f(x, y^{\theta}, z) = (f_{<}, f_m, f_o, f_{fi}, f_s, f_{di}, f_{eq}, f_d, f_{si}, f_f, f_{oi}, f_{mi}, f_{>})^t$.
Consider following situations :




(a)




(b)

$f(x_1, y_1^{\theta}, z) = (f_{<}, f_m, f_o, f_{fi}, f_s, f_{di}, f_{eq}, f_d, f_{si}, f_f, f_{oi}, f_{mi}, f_{>})^t$.
$f(x_2, y_2^{\theta}, z) = (f_{<}, f_m, f_o, f_{fi}, f_s, f_{di}, f_{eq}, f_d, f_{si}, f_f, f_{oi}, f_{oi}, f_{>})^t$.

$f(x_1, y_1^\theta, z) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$.
$f(x_2, y_2^\theta, z) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$.

- $F_{OR}(\theta, A_\theta(v), B_\theta(v) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$.
- $F_{AND}(\theta, A_\theta(v), B_\theta(v) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$.
- $F_{PROD}(\theta, A_\theta(v), B_\theta(v) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$.
- $F_{SUM}(\theta, A_\theta(v), B_\theta(v) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0)^t$.

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**TABLE:** Allen relations and their inverse

| Relation | Inverse |
|---|---|
| < | > |
| m | mi |
| o | oi |
| s | si |
| f | fi |
| d | di |
| = | = |

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| o | oi |
| s | f |
| fi | si |
| d | d |
| di | di |
| = | = |

# Fusion of Topological and directional relations

- $f_E = \sum_{\theta=0}^{\frac{\pi}{4}} A_{r_2} \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} A_{r_1} \times cos^2(2\theta)$
- $f_W = \sum_{\theta=0}^{\frac{\pi}{4}} A_{r_1} \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} A_{r_2} \times cos^2(2\theta)$
- $f_N = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} A_{r_2} \times cos^2(2\theta)$
- $f_S = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} A_{r_1} \times cos^2(2\theta)$

$f \in \{Dsjoint, Meet, Overlap, TPP, NTPP, TPPI, NTPPI, EQ\}$ and $A_1 = \{<, m, o, s, f_i, d, d_i, =\}$, $A_2$ is re orientation of $A_1$.

1. Only first row is non zero then objects have fuzzy disjoint (*DC*) topological relation .

# Approximate a topological relation in 2D

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2. If the first and the second rows are non zero then the overall relation in $2D$ space is fuzzy *meet EC*.

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4. If non zero values also exist in *TPP* along with *NTPP* (*TPPI* along with *NTPPI*) then the relation will be *TPP* (*NTPP*) in the corresponding direction.

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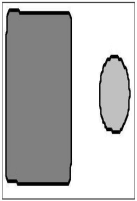
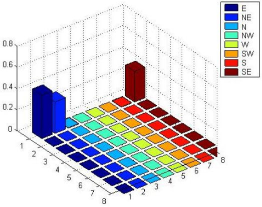
4. If non zero values also exist in *TPP* along with *NTPP* (*TPPI* along with *NTPPI*) then the relation will be *TPP* (*NTPP*) in the corresponding direction.
5. Relations *PP, PPI, EQ* hold if the corresponding relation holds in all directions. A relation will hold if all elements in a row are non zero and all other rows are zero.

# Examples

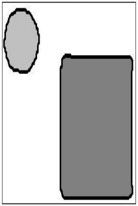
## Disjoint & Meet topological relations



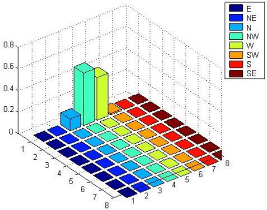
(c)



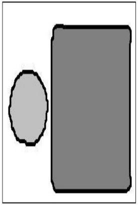

(d)



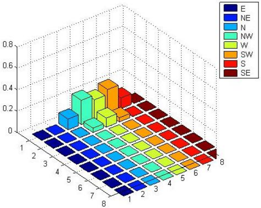
(e)



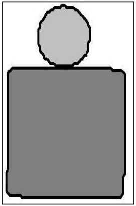

(f)



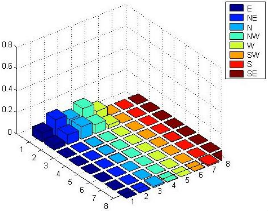
(g)




(h)

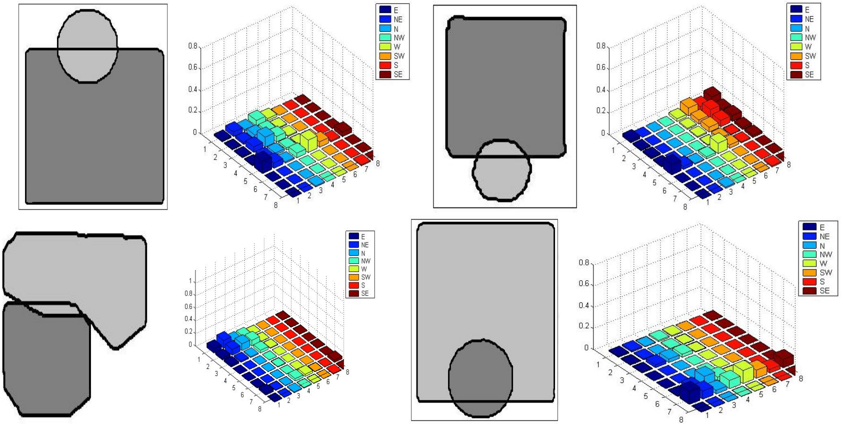


(i)

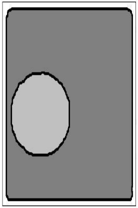



(j)

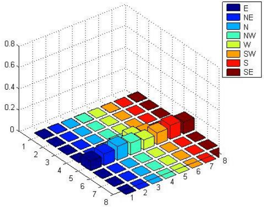
# Examples
## Overlap



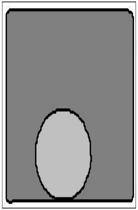
# Examples
## TPP & TPPI



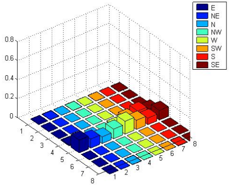
(k)



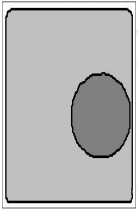

(l)



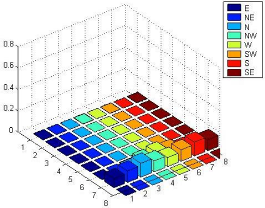
(m)



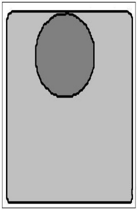

(n)



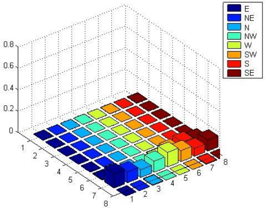
(o)



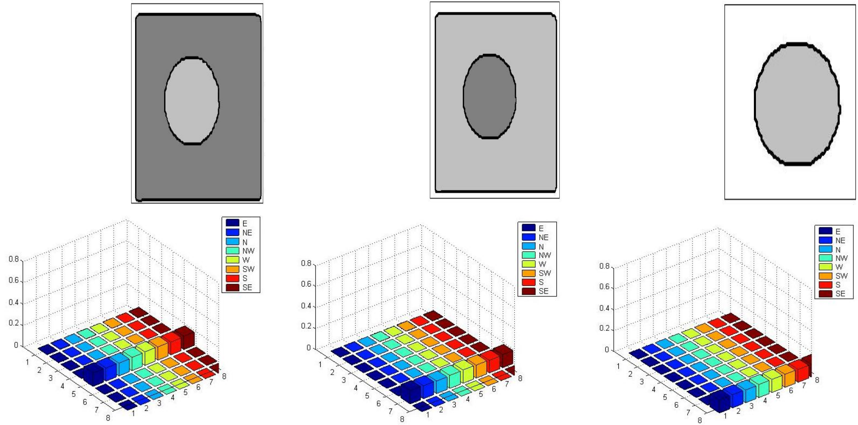

(p)



(q)




(r)



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- Computation time decreases half to the histogram representation.
- Method is fuzzy and can be used to detect the small changes in spatial scene.
- This method can be used for topological and directional predictions.
- Method can be used for fuzzy automatic image interpretation and reasoning.

# Thank you