

# A Novel Hybrid Approach to Improve Performance of Frequency Division Duplex Systems with Linear Precoding

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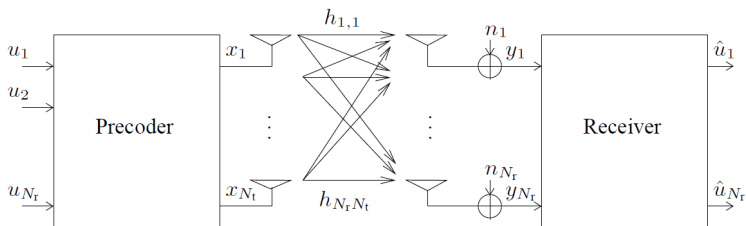
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# Introduction

- ▶ Demand of multimedia contents produces the development of new techniques to improve digital communications throughput.
- ▶ Precoding is an attractive way to remove interferences because it reduces cost and power consumption in the equipment.
- ▶ When using precoding the base station should know the *Channel State Information* (CSI) which is usually acquired at the receiver using supervised algorithms with pilot symbols.
- ▶ In *Frequency Division Duplex* (FDD) systems, used in fixed broadband wireless networks, CSI is sent by means of a feedback channel.
- ▶ In order to reduce the overhead due to the periodical transmission of pilot symbols, we propose to obtain the CSI combining supervised and unsupervised algorithms.

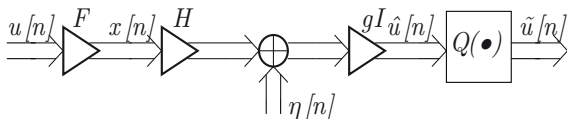
# System Model



- ▶ Flat fading channels with Rayleigh distribution.
- ▶ The channel remains constant during several frames.
- ▶ Gaussian noise  $\boldsymbol{\eta}[n] \sim \mathcal{N}_{\mathbb{C}}(0, \mathbf{C}_{\boldsymbol{\eta}}) \in \mathbb{C}^{N_r}$ .
- ▶ Transmitted signal  $\mathbf{x}[n] \in \mathbb{C}^{N_t}$ .

# Linear Precoding Design: Model

- ▶ System with Precoding over Flat MIMO Channel



- ▶ The data symbols  $\mathbf{u}[n]$  are passed through the precoding filter  $\mathbf{F}$  to form the transmit signal  $\mathbf{x}[n] = \mathbf{F}\mathbf{u}[n] \in \mathbb{C}^N$ .
- ▶ Therefore the received signal is  $\mathbf{y}[n] = \mathbf{H}\mathbf{F}\mathbf{u}[n] + \boldsymbol{\eta}[n]$
- ▶ After multiplying by the received gain we get the estimated symbols  $\hat{\mathbf{u}}[n] = g\mathbf{H}\mathbf{F}\mathbf{u}[n] + g\boldsymbol{\eta}[n]$

## Linear Precoding Design: Wiener Filter

The Wiener filter for precoding is a very powerful transmit optimization that minimizes the MSE with a transmit energy constraint, i.e.

$$\begin{aligned} \{\mathbf{F}_{WF}, g_{WF}\} &= \underset{\{\mathbf{F}, g\}}{\operatorname{argmin}} E \left[ \|\mathbf{u}[n] - \hat{\mathbf{u}}[n]\|_2^2 \right] \\ &\text{s.t.: } \operatorname{trace}(\mathbf{F}\mathbf{C}_u\mathbf{F}^H) \leq E_{tx}. \end{aligned} \quad (1)$$

where  $\mathbf{C}_u = E [\mathbf{u}^H[n]\mathbf{u}[n]]$  and  $E_{tx}$  is the transmit energy. Considering  $g \in \mathbb{R}^+$ , the solution for the Wiener filter is given by

$$\begin{aligned} \mathbf{F}_{WF} &= g_{WF}^{-1} \left( \mathbf{H}^H\mathbf{H} + \xi\mathbf{I} \right)^{-1} \mathbf{H}^H \\ g_{WF} &= \sqrt{\frac{\operatorname{trace} \left( \left( \mathbf{H}^H\mathbf{H} + \xi\mathbf{I} \right)^{-2} \mathbf{H}^H\mathbf{C}_u\mathbf{H} \right)}{E_{tx}}}. \end{aligned} \quad (2)$$

## Neural Network Approaches

- ▶ Our model assumes that the observations are linear and instantaneous mixtures of the user symbols

$$\mathbf{y}[n] = \mathbf{H}\mathbf{F}\mathbf{u}[n] + \boldsymbol{\eta}[n] = \mathbf{A}\mathbf{u}[n] + \boldsymbol{\eta}[n]. \quad (3)$$

- ▶ In order to recover the user symbols, we will use a Neural Network whose output is a combination of the observations

$$\mathbf{z}[n] = \mathbf{W}^H[n]\mathbf{y}[n] = \boldsymbol{\Gamma}[n]\mathbf{u}[n]. \quad (4)$$

where  $\boldsymbol{\Gamma}[n] = \mathbf{W}^H[n]\mathbf{A}$

- ▶ User symbols are optimally recovered when  $\mathbf{W}[n]$  is selected such as every output extract a different single source
- ▶ This occurs when  $\boldsymbol{\Gamma}[n] = \mathbf{D}[n]\mathbf{P}[n]$

## Neural Network Approaches (cont.)

### ▶ Supervised Algorithms:

- ▶ The channel matrix can be estimated minimizing the MSE between  $\mathbf{y}[n]$  and  $\mathbf{x}[n]$
- ▶ Least Mean Squares (LMS) algorithm

$$\mathbf{W}[n+1] = \mathbf{W}[n] - \mu \mathbf{y}[n](\mathbf{W}^H[n]\mathbf{y}[n] - \mathbf{x}[n])^H.$$

- ▶ Wiener-Hopf solution:  $\mathbf{W}[n] = \mathbf{C}_y^{-1}\mathbf{C}_{yx}$   
where  $\mathbf{C}_y = E[\mathbf{y}[n]\mathbf{y}^H[n]]$  and  $\mathbf{C}_{yx} = E[\mathbf{y}[n]\mathbf{x}^H[n]]$ .

### ▶ Unsupervised Algorithms:

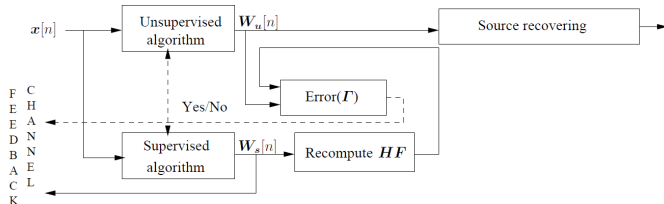
- ▶ Pilot symbols reduce system throughput. Limitation avoided using Blind Source Separation (BSS) algorithms.
- ▶ The source data can be recovered using unsupervised learning algorithms. For example: Infomax algorithm

$$\mathbf{W}[n+1] = \mathbf{W}[n] - \mu \mathbf{W}[n] \left( \mathbf{z}[n] \mathbf{g}^H(\mathbf{z}[n]) - \mathbf{I} \right). \quad (5)$$



## Novel Combined Approach

- ▶ One of the advantages of unsupervised algorithms is their ability to track low channel variations.
- ▶ Supervised algorithms provide a fast channel estimation for low or high variations at the cost of using pilot symbols.
- ▶ We propose to combine these paradigms to obtain similar performance to supervised algorithms with less pilot symbols.
- ▶ *Idea*: determine dynamically the instants where pilot symbols must be transmitted.



## Novel Combined Approach: Procedure

*First frame:*

- ▶ The receiver uses the supervised algorithm to estimate  $\mathbf{H}$ . This matrix is sent to the transmitter by the feedback channel.
- ▶ Both receiver and transmitter compute  $\mathbf{F}$ .
- ▶ The receiver initializes the unsupervised algorithm to  $\mathbf{W}_u[n] = (\mathbf{HF})^{-H} = (\mathbf{A})^{-H}$ .

*Each time a new frame is received:*

- ▶ If the channel has suffered a significant variation send an alarm to the transmitter asking for a "pilot frame". Then the receiver do the same as first frame.
- ▶ If the channel has not suffered a significant variation receiver computes  $\mathbf{W}_u[n]$  and recover the data symbols  $\mathbf{u}[n]$ .

## Novel Combined Approach: Decision Criterion

- ▶ How to determine significant channel variations?
- ▶ The permutation indeterminacy associated to unsupervised algorithms is avoided due to the initialization  
 $\mathbf{W}_u[n] = (\mathbf{H}\mathbf{F})^{-H} = (\mathbf{A})^{-H}$
- ▶ The optimum separation matrix produces a diagonal matrix for  $\mathbf{\Gamma}[n] \Rightarrow$  the mismatch of  $\mathbf{\Gamma}[n]$  with respect to a diagonal matrix allows us to measure the variations in the channel.
- ▶ Dispersion measure:

$$\text{Error}[n] = \sum_{i=1}^{N_t} \sum_{j=1, j \neq i}^{N_t} \left( \frac{|\gamma_{ij}[n]|^2}{|\gamma_{ii}[n]|^2} + \frac{|\gamma_{ji}[n]|^2}{|\gamma_{ii}[n]|^2} \right) \quad (6)$$

- ▶ Decision criteria:

$$\begin{cases} \text{Error}(\mathbf{\Gamma}[n]) > t \rightarrow \text{Use supervised approach} \\ \text{Error}(\mathbf{\Gamma}[n]) \leq t \rightarrow \text{Use unsupervised approach} \end{cases} \quad (7)$$

## Experimental Results: Data

- ▶ We transmit the image *cameraman* (in tif format with 256 gray levels) using a QPSK (8 000 symbols) and a  $4 \times 4$  MIMO system.
- ▶ The channel matrix is updated using the following model

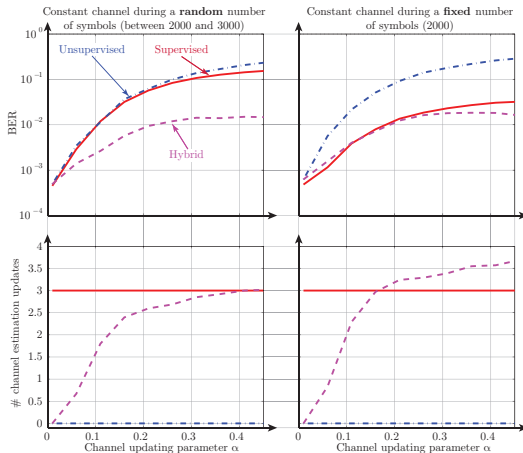
$$\mathbf{H} = (1 - \alpha)\mathbf{H} + \alpha\mathbf{H}_{\text{new}} \quad (8)$$

where  $\mathbf{H}_{\text{new}}$  is a  $4 \times 4$  matrix randomly generated according to a Gaussian distribution.

- ▶ The SNR has been stated to 20 dB.
- ▶ Supervised Wiener-Hopf solution: 200 pilot symbols are transmitted each 2 000 symbols.
- ▶ Infomax algorithm ( $\mu = 0,001$ ) has been initialized to the Wiener-Hopf solution.

# Experimental Results: Performance

- Threshold  $t = 0,7$ , 200 independent realizations.



# Conclusions

- ▶ In order to reduce the overhead due to the transmission of pilot symbols in FDD-LP systems, we have proposed to combine supervised and unsupervised algorithms.
- ▶ The algorithm selection is done by using a simple decision rule which allows to determine the case when the channel has suffered a considerable variation. This information is sent to the transmitter using the feedback channel.
- ▶ The experimental results show that the combined approach is an attractive solution because it provides an adequate BER with a reduced number of pilot symbols.