Percolating Swarm Dynamics

Manuel Graña, Carmen Hernández, Alicia D'Anjou, and Blanca Cases
Computational Intelligence Group, Universidad del Pais Vasco
www.ehu.es/ccwintco

Contents

- Introduction
- Some ideas about percolation
- Boids SI parameters
- Percolation thresholds in Boids SI
- Conclusions

Introduction

- Percolation
  - Physics: slow flow of fluids through a porous media,
  - Math: connectivity among random systems that can be layered over a spatial lattice.
- Deals with the **thresholds** on parameters that set conditions on the **spatial connectivity** and may allow or hinder complete connectivity

Introduction

- Swarm Intelligence has been applied to solve combinatorial problems, i.e. Graph coloring
  - Individuals (boids) move on an space
  - Global spatial configurations are mapped into problem solutions
- Convergence may be related to percolation conditions relating
  - World size
  - Boid's radius of vision
  - Population size

Some ideas about percolation

- Bond Percolation :
  - given a square lattice
  - A link connecting a position (x, y) to an adjacent one (x' , y')
    - Pruned with probability 1 − p,
    - Maintained with probability p,
- Percolation threshold:
  - The the critical value p_c that ensures the connectivity from one side of the lattice to the other for $p \geq p_c$

Some ideas about percolation

- Site Percolation
  - Imagine an electrical potential from one side to the opposite side of the grid.
  - We start to remove the network nodes thereby preventing electrical current flow.
  - What percentage of nodes should be maintained for the current to continue flowing?.
  - The Percolation threshold p_c is the mean of that measure over all the possible grids.

Some ideas about percolation

- At the Percolation threshold, the structure changes from a collection of many disconnected parts to a large aggregate (infinite cluster).
- At the same time, the average size of clusters of finite size that are disconnected to the main cluster, decreases.

Some ideas about percolation

- The probability of a site or link belonging to the infinite cluster is:

$$\theta\left(p\right) \propto \left(p - p_c\right)^{\beta}$$

- In a graph generation process where *p* is the probability of an edge between two nodes on a given n-dimensional lattice.

Boids SI parameters

- Each boid is characterized by
  - Position p_i and velocity v_i
  - Sensorial input: information about boids in an spatial neighborhood of radius *R*

$$\partial_i = \partial(b_i) = \{b_j : \|p_i - p_j\| < R\}.$$

  - Composition of the boid's velocity:

$$v(t+1) = f_{\mathrm{maxv}} \mathcal{N}\left(\alpha_o v(t) + \alpha_s v_s(t) + \alpha_c v_c(t) + \alpha_a v_a(t) + \alpha_n v_n\right) \quad (10)$$

Boids SI parameters

- Separation: steer to avoid crowding local flock-mates inside a private zone of radius *z*.

$$v_{s_i} = - \sum_{b_j \in \partial_i :\, d(b_j, b_i) < z} (p_j - p_i)$$

- Cohesion: steer to move toward the average position of local flock-mates

$$v_{c_i} = c_i - p_i \quad \text{where} \quad c_i = \frac{1}{|\partial_i|} \sum_{b_j \in \partial_i} p_j$$

- Alignment: steer in the direction of the average heading of local flock-mates.

$$v_{a_i} = \frac{1}{|\partial_i|} \sum_{b_j \in \partial_i} v_j - v_i$$

Percolation in Boids SI

- Assumption: Convergence of the Boid SI to some stable needs that all the boids are connected through their sensory input.
- Percolation conditions relate
  - The size of the world (square torus) *S*
  - The radius and area of vision $A = \pi R^2.$
  - The population of boids $\mathcal{P}$

Percolation in Boids SI

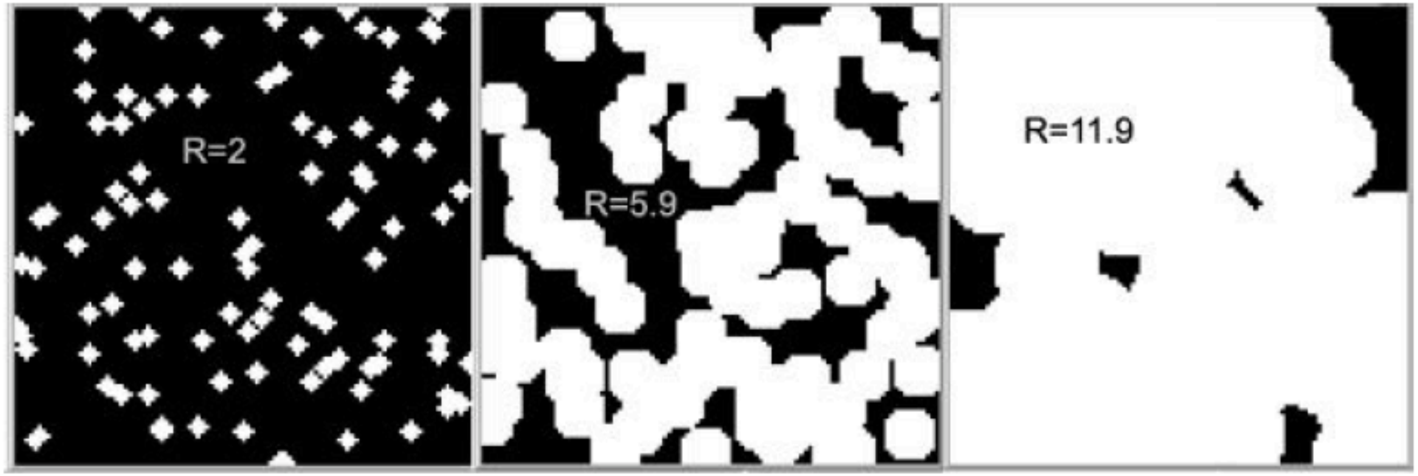

Fig. 1. Spatial neighborhoods of randomly positioned boids for various values of R. Largest radius ensures spatial connectivity.

- Numerical simulation:
  - Generate random positions of the population boid and study the % of connected area

$$a = \lambda A = \lambda \pi R^2 = \frac{\mathcal{P}}{S^2} \pi R^2,$$

  - Under the assumption of a Poisson distribution of the number of boids per unitary patch

$$P_\lambda (n) = P(\mathcal{P} = n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

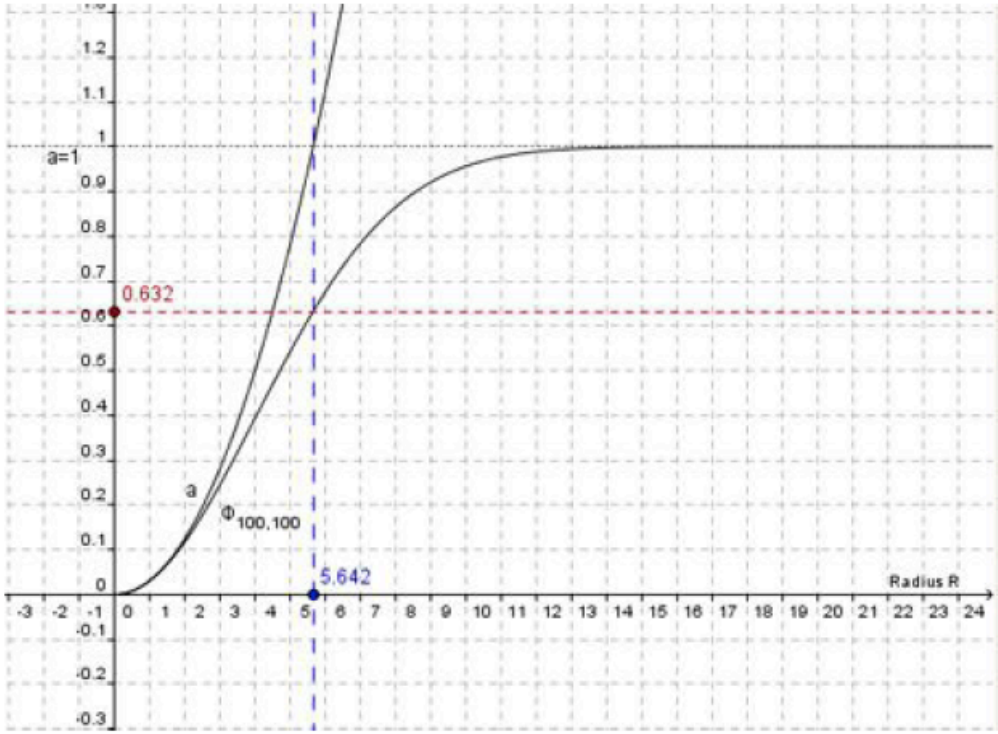

Fig. 2. Percolation: the connected area a as a function of R compared to $\Phi_{100,100}$

- Where the following normalization is used

$$\Phi_{P,S}(R) = 1 - e^{-\frac{P}{S^2}\pi R^2}$$

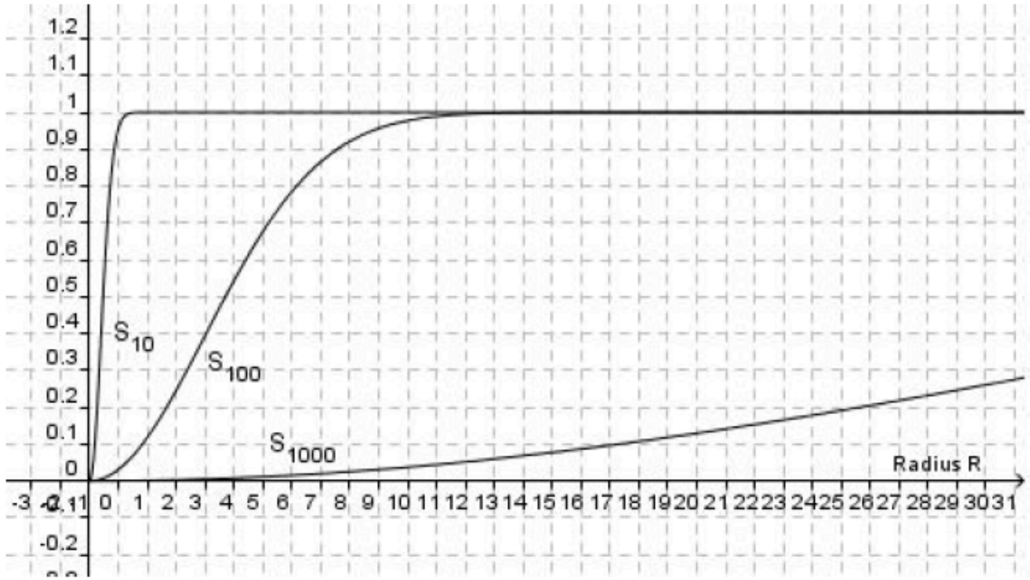

Fig. 3. $\Phi_{P,S}$ for population of $\mathcal{P} = 100$ boids in a square of side $S = 10{,}100$ and 1000 patches

- Probability of an infinite component at the origin

$$\theta(R, \lambda) = \theta(a)$$

$$0 \leq \theta(a) \leq 1$$

- 0-1 law of Kolmogorov: there is a critical area

such that $\theta(a) = 0$ for $a < a_c$ and $\theta(a) > 0$ for $a > a_c$

$$\theta(a_c) = p_c$$

- For an hexagonal lattice we find:

$$2.184 \leq a_c \leq 10.588$$

- Which can be mapped into radius bounds

$$R\left(a\right) = \sqrt{\frac{aS^2}{\pi P}} \Rightarrow 8.338 \leq R_c \leq 18.358$$

$$\mathcal{P} = 100 \quad S = 100$$

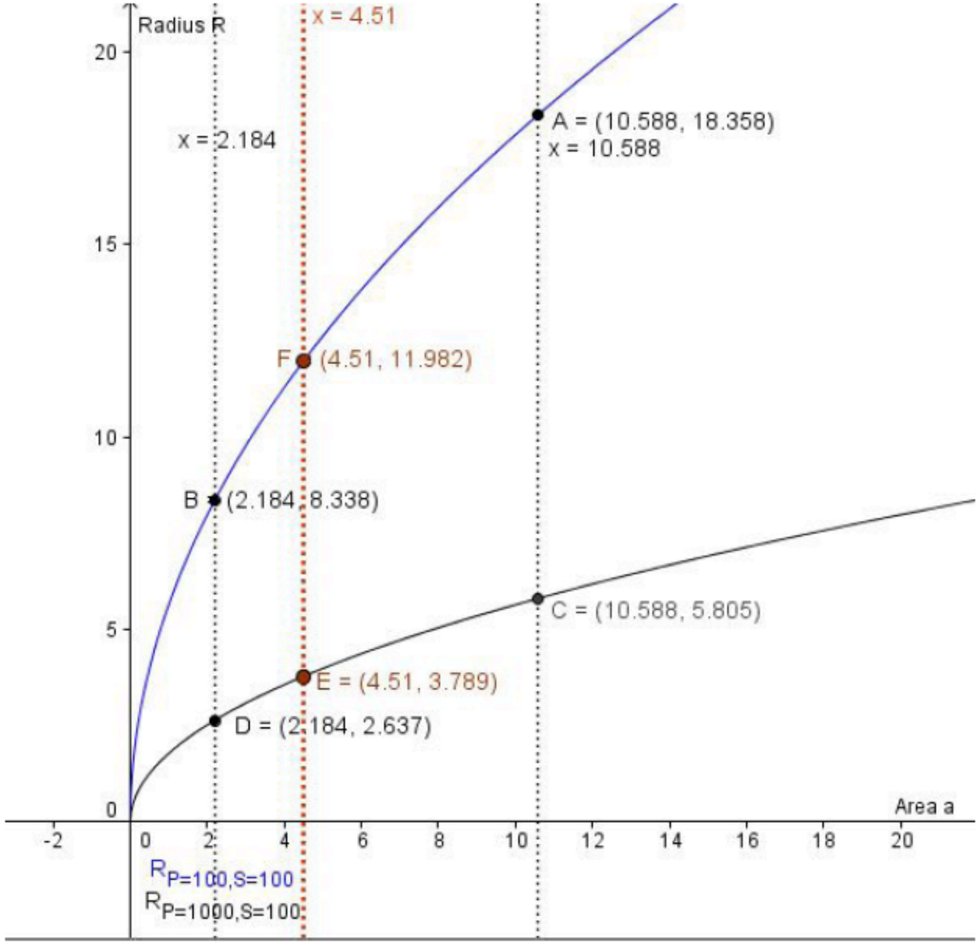

Fig. 4. Variation of the boid's radius of perception versus the area

Conclusions

- Percolation can be applied to the study of the convergence of Boids Swarm systems
  – Connectivity of the entire swarm is a precondition for convergence
  – We have derived bounds on the radius for specific cases.